NATIONAL HIGHER SCHOOL OF ENGINEERING YAOUNDE

Course support

DIGITAL IMAGE PROCESSING

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TABLE DES MATIERES

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CHAPTER1. FUNDAMENTALS ON DIGITAL IMAGE

1.1 Objectives and learning outcomes

The goal of this chapter is to present the context and basic notions on digital image processing, and remote sensing images in particular. By the end of the chapter, every student must be able to:

1. Define and characterise a digital image;
2. Define and characterise a pixel;
3. Define and characterise image sensors;
4. Present the digital image acquisition principle;
5. Compare an 8-bits with a 16-bits format image;
6. Compare a monoband with a multiband image;

1.2 Definition

In general, an image is an information carrier. It has the elements of a scene that was captured by either a camera or a satellite. Remote sensing images are generally derived from onboard satellite sensors, and sometimes aerial photographs. An image can have different definitions depending on the context. In signal processing, an image is defined as a two-dimensional signal. Mathematically, an image is a real application IMG defined as follows:

$$IMG \quad : \quad (M \times N) \subset IR \times IR \longrightarrow X \subset IR \tag{1.1}$$

In case of digital images, the subset (MxN) consists of pairs of integers (x,y), with $x \in \{0,1,2.....NC\}$, and $y \in \{0,1,2.....NL\}$, where NL and NC represent respectively the number of lines and columns of the image. So, a digital image is finally a matrix made of NL lines and NC columns. Each cell of this matrix is an object point called **pixel**, acronym of "picture element", and the coordinates of the cell are the coordinates of the object point in the observed region.

1.3 Digital image acquisition principle

1.3.1 Principle

Three components are involved in the digital image acquisition principle:

- The object for which the image is taken;
- A source that generates radiations;
- A sensor that observes the object.

The following figure illustrates the digital image acquisition principle.

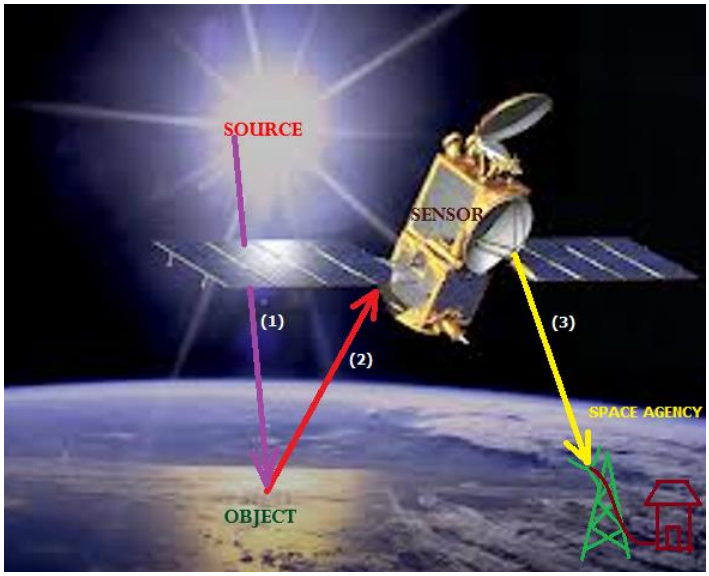


Figure 1.1. Digital Image acquisition principle

(1): The source, that can be the Sun, in case of an optical remote sensing image, illuminates the object with electromagnetic radiations;

(2): The object, that can be a region of the Earth, reflects totally or in part the received radiation towards the sensor;

(3): The Sensor, that can be on-board of a satellite, quantifies the received radiation from the object and sends the value to the Space agency for registration.

During the process of quantification, if the maximum value a radiation can take is 255, we talk of 8-bits image and each pixel of the image is coded on one byte. In case a radiation value can reach 65535, we talk of 16-bits image and each pixel of the image can be coded on 2 bytes.

The space Agency is equipped with a magnetic band which registers sequentially and progressively data sent by the sensor in the form of bytes. This registration is done pixel after pixel and line after line.

1.3.2 Sensors functioning

A sensor is an equipment that possesses an analogical to numerical and/or a numerical to analogical converter of signals. Each sensor is sensitised to a certain range of signal frequencies characterised by a minimum value f_{min} called low frequency and a maximum value f_{max} called high frequency of the sensor. Only the radiations with frequencies between f_{min} and f_{max} are detectable by this sensor. The frequency difference $\Delta f=f_{max}-f_{min}$ is called **spectral resolution** of the sensor. The smaller this difference is, the higher the spectral resolution is.

The sensor is made of a certain number of synchronised detectors with the same spectral resolution. Each detector observes a specific point on the object. This point of a certain surface is called **pixel** and its superficies is called **spatial resolution**. At a time (clock – top), one line of the image is taken by these detectors and registered. So, an image of NL lines is taken in NL clock-tops. If the sensor has NC detectors, then the image has NC columns. In fact, each detector observes one column of the image. This is why a digital image is finally a matrix.

The following table presents some sensors with their characteristics.

Table 1.1 Examples of sensors with some characteristics.

Sensor	Lunching date	Altitude	Spatial Resolution	Dimension of the Scene
LANDSAT 5 (USA)	March 1985	705 km	30 x 30 m²	185 x 172 km²
SPOT (France)	February 1986	822 km	20 x 20 m² (10 x 10 m in Panchromatic mode)	60 x 60 km²
RADARSAT (Canada)	November 1995	793 km	10, 25, 50 et 100 m	50 à 500 km²
ERS (Europ)	July 1991	785 km	25 x 25 m² (12.5 x 12.5 in PRI mode)	100 x 100 km²
JERS (Japan)	February 1992	568 km	18 x 24 m²	75 x 75 km²
ESAR (USA)	1995	8 km	6 x 6 m²	

The following table presents some sensors observing the Cameroon Country.

Table 1.2 Examples of sensors observing Cameroon

Sensor	Frequency or wavelength	Sensor Type	Passing Frequency	Disponibility in Cameroon
LANDSAT 5 (USA)	0.76 – 0.90 µm (IR) 1.55 – 1.75 µm (IRM) 10.4 – 12.5 µm (IRT) 2.08 – 2.35 µm (IRM)	Thematic Mapper (TM) (scanning radiometer)	16 days	Partially (North Region)
SPOT (France)	0.50 – 0.59 µm (B) 0.61 – 0.68 µm (V)	HRV (High Resolution Visible)	3 to 26 days	Partially (North Region)

	0.79 – 0.89 µm (IR) 0.51 – 0.73 µm (Panchromatic)	(scanning radiometer)		
RADARSAT (Canada)	5.3 Ghz (Bande C) 5.66 cm Polarisation HH	**RSO** (active sensor)	16 days (3 days in Canada)	Total
ERS (Europ)	5.3 Ghz (Bande C) 5.66 cm Polarisation VV	**RSO** (active sensor)	35 days	Total
JERS (Japon)	0.52 – 0.60 µm (B) 0.63 – 0.69 µm (V) 0.76 – 0.86 µm (R) 0.76 – 0.86 µm (IR) 1.60 – 1.71 µm (IRM) 2.01 – 2.12 µm (IRM) 2.13 – 2.15 µm (IRM) 2.27 – 2.40 µm (IRM) 5.3 Ghz (Bande C) 5.66 cm Polarisation VV	SOP (Passive Sensor) SAR (Active Sensor)	44 days	Partially Localised (Kribi Region)
ESAR (USA)	5.3 Ghz (Band C) 5.66 cm Polarisation VV	**SAR** (Active Sensor)		Localised (Douala Region)

1.4 Digital image representation

As previously developed, a digital image is a matrix of integers. Each cell of the matrix contains a value called greyscale of the related pixel. The following figure 1.2 presents an example of a greyscale image.

Figure 1.2 Portion of a 375 x 200 pixels (375 columns and 200 lines) of a greyscale image.

This image is an 8-bits format image, acquired by the ERS-1 sensor, with 12.5m side spatial resolution, on the Cameroonian Atlantic coast. Each greyscale represents a level of grey between black and white colours. The value 0 corresponds to the black colour and the value 255 corresponds to the white colour. In remote sensing, one associates to each

greyscale or range of greyscales a particular colour. This process is done by running an algorithm similar to the following.

Algorithm 1.1 Image conversion from grayscales to colours

Input: Img: digital image;

T[1,..n]: Array of n thresholds of greyscales;

C[1,..n]: Array of n colours;

Output: Ima: coloured image (matrix of colours);

BEGIN

FOR each pixel (x,y) of the image Img, **DO**

IF $0 \le \mathrm{Im}g(x,y) \le T[1]$ **THEN** Assign the colour C[1] to the cell Ima(x,y);

ELSEIF $T[1] < \mathrm{Im}g(x,y) \le T[2]$ **THEN** Assign the colour C[2] to the cell Ima(x,y);

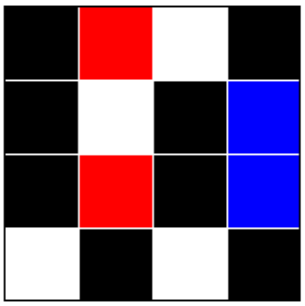
ELSEIF $T[n-1] < \mathrm{Im}g(x,y) \le T[n] = 255$ **THEN** Assign the colour C[n] to the cell Ima(x,y);

END;

The following figure gives an illustrative example, considering the thresholds array T=[50,100,150] and the colours array C=[Black, Red, Blue, White].

0	100	200	17
15	240	9	150
11	70	37	110
255	40	190	7

(a)



(b)

Figure 1.3 (a) Portion of a digital image of size 4x4; (b) coloured representation of the image according to the Algorithm 1.1.

1.5 Header files

Most of the time, greyscales of images are registered sequentially on a medium such as CD, DVD or USB device. It therefore becomes impossible to display the image without knowing complementary information such as the number of lines and columns, the number of bands and the image format (pixels of some images are coded on one byte whereas pixels of other images are coded on 2-bytes). This is why digital image files are

always coupled with an additional file called **header file**. This file contains complementary information required to display, analyse and interpret the image. Among others, following are data contained in a header file: Orbital number, date of acquisition, number of bands, image format (8-bits or 16-bits image), the number of columns and lines, the geographical coordinates of the scene, the polarisation, the spectral and spatial resolution, the name of the sensor, the altitude, etc. For a compressed image such as BMP, JPG, GIF, PNG, TIF, GeoTiff etc., The header file and the image data are merged into a single file in which a certain number of bytes generally at the beginning of the file are reserved essential information of the header file.

1.6 Mono band and multi band images

On-board of some spatial engines (Satellite, Planes) are embarked many sensors, each sensor possessing its proper spectral resolution, but all of them synchronised to take the picture of the same object at the same moment. At the end of the acquisition process, for a system of N sensors, the resulting image is made of N **bands** or **channels**. A band or channel is the image of an object acquired by a sensor in a multi sensors system. In other words, it is the image of an object acquired under a certain wavelength. A **mono-band** or **mono-channel** image is then an image acquired by a system made of a single sensor. ERS-1 or ERS-2 is an example of such a system. A **multi-band** or **multi-channel** image is an image acquired by a system made of several sensors sensitised to different frequencies. This is the case for example of Landsat5 and SPOT. The following illustrates a 3-bands image acquired by the SPOT system.

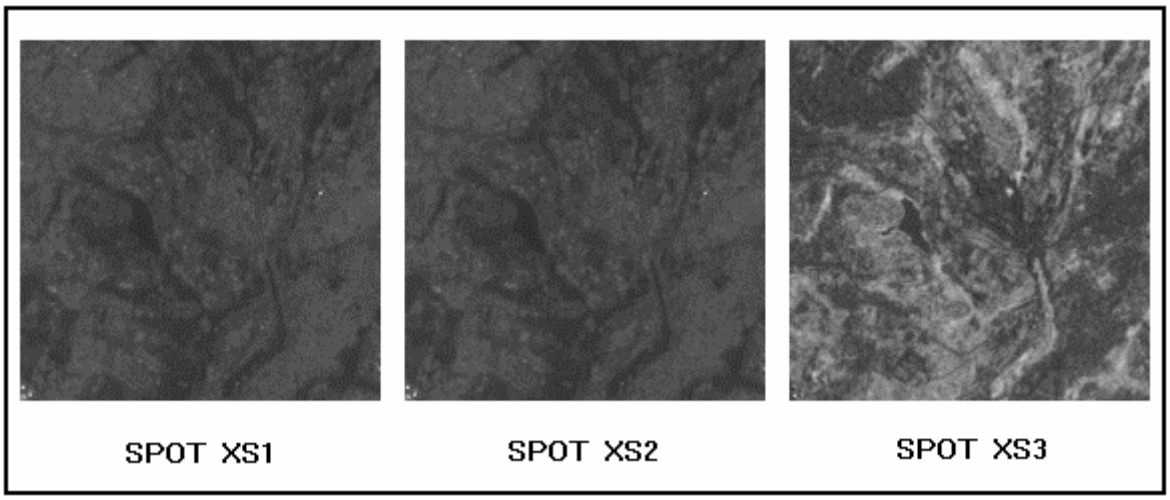


Figure 1.4 A portion of size 180x180 pixels of a multi-band acquired by SPOT

This image of 20m size spatial resolution is part of Yaounde headquarters.

In fact, within a certain range of wavelength, two different objects can give the same spectral answer. For example, a lake and a road can give the same spectral value if they are observed under a certain wavelength. By changing the wavelength of sensor sensibility, they can be discriminated. This is why a multi-band image gives more information about the observed image than the mono-band image. But this fact has nothing to do with the accuracy of the image which is improved as the spatial resolution is higher.

Mathematically, a multi-band image can be defined as follows:

$$\begin{array}{lll} IMG & :(M\times N)\subset IN\times IN & \longrightarrow \quad IR^n \\ & (x,y) & \mapsto \quad (I_1(x,y),I_2(x,y),....,I_n(x,y)) \end{array} \qquad (1.2)$$

In this equation, $I_k(x,y)$ is the answer of the pixel at location (x,y) of the object to the k^{th} sensor of the system. Each pixel of the object has exactly n values.

At a time (clock-top), each sensor take the first line of the observed object and sends for registration. i.e. if the image is made of NC columns then the first NC pixels of the magnetic band constitute the first line of the first band; the NC following pixels constitute the first line of the second band and progressively till the first line of the last band and the process continues with the second line of the first band and similarly till the last line of the last band.

In case of an 8-bits image, each byte of the magnetic band or medium represents one pixel. But in case of a 16-bits image, each pixel is represented on the medium by a couple of bytes.

Example: Let consider the following magnetic band representing a 4-band 16-bits image, made of NC=4 columns and NL=2 lines.

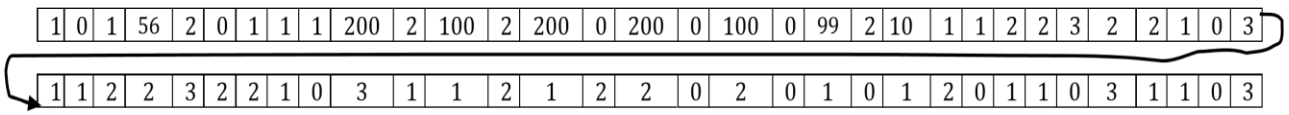


Figure 1.5. A magnetic band

NB: The second line follows the first.

Solution:

Since it is a 16-bits image, each pixel is code on the magnetic band by a couple of bytes. So the magnetic band which contains 64 bytes is made of 32 pixels.

Each band having 4 columns and two lines, each line is made of 4 pixels, i.e. 8 bytes. So the first 8 bytes constitute the first line of the first band (Band A), the following 8 bytes

constitute the first line of the second band and progressively, we obtain the following extracted bands:

Band A	1	0	1	56	2	0	1	1	1	1	2	2	3	2	2	1
Band B	1	200	2	100	2	200	0	200	0	3	1	1	2	1	2	2
Band C	0	100	0	99	2	10	1	1	0	2	0	1	0	1	2	0
Band D	2	2	3	2	2	1	0	3	1	1	0	3	1	1	0	3

Figure 1.6. Extracted bands

1.7 Exercises

- What is a digital image?
- What is the principle of image acquisition?
- What is the spectral resolution of an image?
- What is the spatial resolution of an image?
- What is the difference between a spatial resolution and a spectral resolution?
- How are image data stored on a medium?
- What is the header file? What is its usefulness?
- What is a mono-band image?
- What is a multi-band image?
- What is the usefulness of a multiband image?
- Let consider the following image presented on figure 1.7.
  - Draw the equivalent image on a magnetic band;
  - Convert this image into a coloured image made of 5 colours, with thresholds: 50, 100, 150, 200 and 255.

10	99	100	255	200	180	55
78	87	23	45	33	77	90
0	55	23	67	88	114	45
77	99	92	90	87	70	88
159	95	200	77	0	0	63
87	74	0	11	74	14	43
44	77	98	49	84	45	23

Figure 1.7 A digital image of size 7x7 pixels.

- Let consider the magnetic band of figure 1.6 of the previous example.
  - Extract the various bands, considering that instead of 4 columns and 2 lines, it is now 2 columns and 4 lines;
  - Extract the various bands, considering that instead of 4 columns and 2 lines, it is now 1 column and 8 lines;
  - Extract the various bands, considering that instead of 4 columns and 2 lines, it is now 8 columns and 1 line;
  - Extract the various bands, considering that, instead of 4 bands, it is now 2 bands and instead of 4 columns and 2 lines, it is now 4 columns and 4 lines.

CHAPTER 2. READING AND DISPLAYING AN IMAGE

2.1 Objective

This chapter aims to present various techniques of reading digital images from a medium (Hard disk, CD, DVD…) and displaying on a region of a screen. The displaying can only be possible when essential header file information such as the format of the image, and the number of bands, lines and columns are known. In this chapter, we suppose that the number of columns and lines of the image are known. By the end of this chapter, every student should be able to:

- Present the principle of reading and displaying an 8-bits format image pixel by pixel;
- Write an algorithm that reads, from a medium, an 8-bits format image and displays on a given region of the computer screen pixel by pixel;
- Present the principle of reading and displaying an 8-bits format image line by line;
- Write an algorithm that reads, from a medium, an 8-bits format image and displays on a given region of a computer screen line by line;
- Present the principle of reading and displaying an 8-bits format image globally;
- Write an algorithm that reads, from a medium, an 8-bits format image and displays on a given region of a computer screen globally;
- Present the principle of reading and displaying a zone of interest on an 8-bits format image;
- Write an algorithm that reads a region of interest from a medium and displays on a given area of the computer screen;
- Present the principle of converting a digital image from a 16-bits format (in intensities or in amplitude) into an 8-bits format image;
- Write an algorithm that converts a digital image from a 16-bits format (in intensities or in amplitude) into an 8-bits format image;
- Program all the previous algorithms in any programming language (Java, C++Builder, Php, Matlab).

2.2 Reading and displaying 8-bits format images

The following figure illustrates the data storage mode, applied on the digital image of figure 1.7.

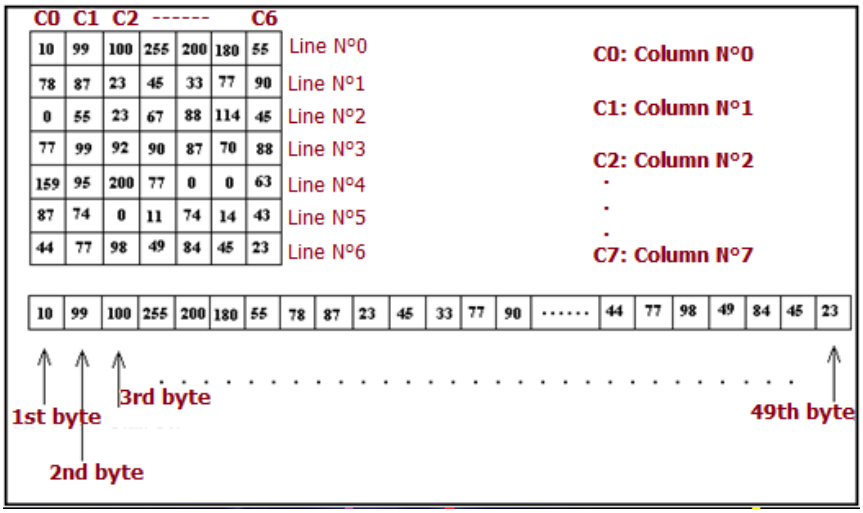


Figure 2.1 Sequential representation of bytes of a digital image on a magnetic band.

Greyscales are stored on the magnetic band or data support (medium) sequentially, starting from the first pixel of the first line till the last pixel of the last line. Once the principle mastered, it is now easy to design an algorithm that reads an image from the magnetic band and displays on a computer screen. In the following sections, we present three methods of reading and displaying digital images. These methods are:

- Pixel by pixel method;
- Line by line method and
- Global method.

In the following algorithms, the magnetic band is assimilated to a file of bytes.

2.2.1 Pixel by pixel method

This method consists in opening the file of bytes (image file), reading bytes successively and displaying them as they are red. This method, in comparison with others is slow. This is why it is no more implemented nowadays in most digital image processing softwares. The strength of this method is that it requires les material (memory space). But the material problem is no more up to date. The related algorithm is the following.

Algorithm 2.1 ReadDisplay_8bits_Pixel_By_Pixel
Input:
NL: Number of lines of the image;
NC: Number of columns of the image;
FileName: Name of the image file (File of bytes);
Local variables:
x , y: **integer** ;//Location of a pixel
Pix: Byte; // *to store temporarily each byte of the image*
BEGIN
- Open the image file named **FileName**;
- **FOR** y varying from 0 to **NL**-1 **DO**
  FOR x varying from 0 to **NC**-1 **DO**
  BEGIN
  -Read one byte in the image file and store in **Pix**;
  -Display the colour corresponding to the value of Pix at the location (x,y) of the screen;
  END
- Close the image file;

END.

2.2.2 Line per Line method

This method consists in reading at once a line of the image and displaying the corresponding colours. This process is repeated till the image is entirely read. For this purpose, a variable of type Array of bytes is declared to store one line of the image each time. The related algorithm is the following.

Algorithm 2.2 ReadDisplay_8bits_Line_per_Line
Input:
NL: Number of lines of the image;
NC: Number of columns of the image;
FileName: Name of the image file (File of bytes);
Local variables:
x , y: **integer** ;//Location of a pixel
LB: Array of NC Bytes; // *to store temporarily each line of bytes of the image*
BEGIN
- Open the image file named **FileName**;
- **FOR** y varying from 0 to **NL**-1 **DO**
  BEGIN
  -Read one line of bytes in the image file and store in **LB**;
  FOR x varying from 0 to **NC**-1 **DO**
  -Display the colour corresponding to the value of LB[x] at the location (x,y) of the screen;
  END
- Close the image file;

END.

The difference with the pixel by pixel method is that, instead of a single pixel, an entire line of the image is read each time the image file is accessed. Experimentally, it has been noticed that this method is at least twice faster than the pixel by pixel one.

2.2.3 Global method

This method consists in reading at once the entire image and displaying the image at the relevant location. For this purpose, a variable of type Image is declared to store temporarily the image during the process. This method is faster than the previous ones. This is why it is used in most digital image processing softwares. The related algorithm is the following.

Algorithm 2.3 ReadDisplay_8bits_Global
Input:
NL: Number of lines of the image;
NC: Number of columns of the image;
FileName: Name of the image file (File of bytes);
Local variables:
x , y: **integer** ;//Location of a pixel
Bitmap: Array [0 .. NL-1, 0 .. NC-1] of Bytes; // *to store temporarily the entire image*
BEGIN
Open the image file named **FileName**;
FOR y varying from 0 to **NL**-1 **DO**
FOR x varying from 0 to **NC**-1 **DO**
Read one byte in the image file and store in **Bitmap[y,x]**;
FOR y varying from 0 to **NL**-1 **DO**
FOR x varying from 0 to **NC**-1 **DO**
Display the colour corresponding to the value of Bitmap[y,x] at the location (y,x) of the screen;
Close the image file;
END.

2.3 Reading and displaying a zone of interest in an 8-bits format images

Sometimes, we are interested in reading and displaying just a portion of an image. This portion is called Zone of Interest (ZOI). It is characterised by a starting point and an ending point as shown in the following figure. In this case, the coordinates of the ending point of the ZOI should be identified during the reading process of the image file.

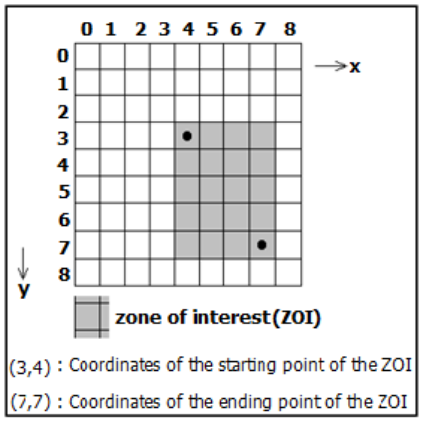


Figure 2.2. Illustration of a zone of interest (ZOI) on a digital image.

In particular, one should notice that the file reader should be positioned on the byte corresponding to the starting point of the ZOI. This starting point is the byte N° $(Y_0*NC)+X_0+1$, (X_0, Y_0) being the coordinates of the starting point of the ZOI. The ZOI is an image of X_1-X_0+1 columns and Y_1-Y_0+1 lines, (X_1, Y_1) being the coordinates of the ending point of the ZOI. The reading and displaying of a ZOI can be done using one of the previous methods.

2.3.1 Reading and displaying a ZOI using the Pixel by pixel method

Following is the algorithm of reading and displaying a ZOI using the pixel by pixel method:

Algorithm 2.4 ReadDisplay_ZOI_8bits_Pixel_By_Pixel
Input:
NL: Number of lines of the image;
NC: Number of columns of the image;
FileName: Name of the image file (File of bytes);
(X_0, Y_0), (X_1, Y_1): respectively the coordinates of the starting and ending points of the ZOI;
Local variables:
x , y: **integer** ;//Location of a pixel
Pix: Byte; // *to store temporarily each byte of the image*
BEGIN
Open the image file named **FileName**;
FOR y varying from Y_0 to Y_1 **DO**
BEGIN
Place the file reader on the byte N° (y*NC)+Xo+1 in the image file;
FOR x varying from 0 to X_1-X_0 **DO**
BEGIN
Read one byte in the image file and store in **Pix**;
-Display the colour corresponding to the value of Pix at the location $(y-Y_0, x)$ of the screen;
END
Close the image file;
END.

2.3.2 Reading and displaying a ZOI using the Line by Line method

Following is the algorithm of reading and displaying a ZOI using the Line by Line method:

Algorithm 2.5 ReadDisplay_ZOI_8bits_Line_By_Line
Input:
NL: Number of lines of the image;
NC: Number of columns of the image;
FileName: Name of the image file (File of bytes);
(X_0, Y_0), (X_1, Y_1): respectively the coordinates of the starting and ending points of the ZOI;
Local variables:
x , y: **integer** ;//Location of a pixel
LB: **Array** of X_1-X_0+1 Bytes; // *to store temporarily each line of the ZOI*
BEGIN
Open the image file named **FileName**;
FOR y varying from Y_0 to Y_1 **DO**
BEGIN
Place the file reader on the byte N° (y*NC)+Xo+1 in the image file;
Read X_1-X_0+1 bytes from the image file and store in LB;
FOR x varying from 0 to X_1-X_0 **DO**
Display the colour corresponding to the value of LB[x] at the location (y-Y_0, x) of the screen;
END
Close the image file;
END.

2.3.3 Reading and displaying a ZOI using the global method

Following is the algorithm of reading and displaying a ZOI using the global method:

Algorithm 2.6 ReadDisplay_ZOI_8bits_Global
Input:
NL: Number of lines of the image;
NC: Number of columns of the image;
FileName: Name of the image file (File of bytes);
(X_0, Y_0), (X_1, Y_1): respectively the coordinates of the starting and ending points of the ZOI;
Local variables:
x, y: **integer** ;//Location of a pixel in the ZOI
Bitmap: **Array** [0..Y_1-Y_0; 0..X_1-X_0] of Bytes; // *to store temporarily the entire ZOI*
BEGIN
Open the image file named **FileName**;
FOR y varying from Y_0 to Y_1 **DO**
BEGIN
Place the file reader on the byte N° (y*NC)+Xo+1 in the image file;
FOR x varying from 0 to X_1-X_0 **DO**
Read one byte from the image file and store in Bitmap[y-Y_0, x];
FOR y varying from 0 to Y_1-Y_0 **DO**
FOR x varying from 0 to X_1-X_0 **DO**
Display the colour corresponding to the value of Bitmap[y,x] at the location (y,x) of the screen;
Close the image file;
END.

2.4 Reading and displaying 16-bits format images

In fact, a human eye cannot distinguish more than 256 colours (greyscales) between the black and white colours. Since 16-bits images have more than 256 grayscales (65536 grayscales), it is therefore necessary to convert into an 8-bits image for displaying purpose. After conversion, one of the three previous methods can be used to display the image.

2.4.1 Conversion of images from 16-bits to 8-bits format.

Pixels of a 16-bits image have grayscales coded in the form of a complex number **a+ib**, where a and b are integers varying from 0 to 255 each. Generally, one distinguishes two categories of 16-bits images:

- 16-bits images in amplitudes;
- 16-bits images in intensities.

For images in intensities, the value of the grayscale **Pix** of a pixel is the intensity of the complex number related to the pixel ($Pix=a^2+b^2$). This number can be greater than 255. This is why it cannot be coded on a single byte but on two.

For images in amplitudes, the grayscale **Pix** of a pixel is the squawroot of the intensity of the related complex number. It is also possible to obtain a 16-bits in amplitude from the averages of complex number amplitudes. Grayscales of such images are already coded on 8-bits. In the following sections, we present the methods of converting each type of 16-bits image into an 8-bits image.

2.4.1.1 Conversion of 16-bits images in amplitude

To convert a 16-bits image in amplitudes into an 8-bits image, some statistics are required. Considering the image **Img** as a matrix of NL lines and NC columns, the Average **(AVG)**, the average of squares **(AvgSq)** and the Standard Deviation **(SD)** of data contained in the matrix are given by the following equation.

$$\begin{cases} AVG = \frac{1}{NL \times NC} \sum_{y=0}^{y=NL-1} \sum_{x=0}^{x=NC-1} Img[y,x] \\ AvgSq = \frac{1}{NL \times NC} \sum_{y=0}^{y=NL-1} \sum_{x=0}^{x=NC-1} (Img[y,x])^2 \\ SD = \sqrt{AvgSq - (AVG)^2} \end{cases} \quad \text{(Eq. 2.1)}$$

The following algorithm converts an image from 16-bits in amplitudes to 8-bits.

Algorithm 2.7 Conversion16_8bits_amp
Input:
NL, NC: integers; //Respectively the number of lines and columns of the image;
FileName: String; //Name of the 16-bits image file (file of bytes)
Local Variables:
x,y: integers ; // Coordinates of a pixel in the image
LB: Array[1,--,2NC] of bytes; // Line of bytes
IMA: Array[0,--,NL-1;0,--,NC-1] of reals; // to store temporarily the matrix image
Output
IM8: Array[0,--,NL-1; 0,--,NC-1] of bytes; // Converted image
BEGIN
Open the image file named FileName;
FOR y varying from 0 to **NL**-1 **DO**
BEGIN
Read 2NC bytes in the image file and store in LB;
FOR x varying from 0 to **NC**-1 **DO**
IMA[y,x] ← **LB**[2*x]*256+LB[2*x+1];
END
Calculate the statistics (AVG, AvgSq and SD) on data of the matrix IMA;
FOR y varying from 0 to NL-1, **DO**
FOR x varying from 0 to NC-1, **DO**
IM8[y,x] ← **minimum{ENT(**(255*IMA[y,x])/(**AVG** + 3***SD**)**), 255}**;
Close the image file *FileName*;
END

ENT represents the entire part.

Once converted from 16-bits to 8-bits, one can use any of the three basic methods to read and display the converted image. Sometimes, we are not interested in the entire image, but just in a portion of a 16-bits image. How do we process in this case?

2.4.1.2 Reading and displaying a zone of interest in a 16-bits images in amplitudes

Reading and displaying a zone of interest of a 16-bits image is similar to the case of an 8-bits image. Following is the related algorithm.

Algorithm 2.8 Read_Display_ZOI_16bits_Amplitudes
Input:
NL, NC: integers; //Respectively the number of lines and columns of the image;
FileName: String; //Name of the 16-bits image file (file of bytes)
(X_0,Y_0), (X_1,Y_1): resp. the coordinates of the starting and ending pixels of the ZOI;
Local Variables:
x,y: integers ; // Coordinates of a pixel in the image
a,b: integers;
IMA: Array[0,--,X_1-X_0;0,--,X_1-X_0] of reals; // to store temporarily the matrix image
Outputs*:* x ,y: de type **entier** ;
IM8 : Array[0,--,Y_1-Y_0;0,--,X_1-X_0] of bytes;
BEGIN
Open the image file named *FileName*;
FOR y varying from Y_0 to Y_1, **DO**
BEGIN
Place the file reader on the byte N° 2*(y*NC+X_0);
FOR x varying from 0 to X_1-X_0, **DO**
BEGIN
Read 2 bytes from the image file and store respectively in a and b;
IMA[y-Y_0,x]←a*256+b;
END
END
Calculate the statistics (AVG, AvgSq and SD) on data of the matrix IMA;
FOR y varying from 0 to Y_1-Y_0, **DO**
FOR x varying from 0 to X_1-X_0 **DO**
IM8[y,x] ← **minimum{ENT(**(255*IMA[y,x])/(**AVG** + 3***SD**)**), 255}**;
Close the image file *FileName*;

IM8 contains the converted image that can be displayed using any of the three methods developed above.

2.4.1.3 Conversion of 16-bits images in intensities (PRI images)

To convert an image from 16-bits in intensities to 8-bits, one should remember that the value on 8 bits of each grayscale is a function of two consecutive bytes (upper and lower bytes) in the image file. Before calculating the various statistics on the image, one should first do the following transformation.

$$G_8 = [(256 * B_u) + B_l]/3 \qquad \text{(Eq. 2.2)}$$

Where G_8 is the grayscale of the pixel on 8-bits; B_u and B_l are respectively the upper-byte of the lower-byte of the pixel. The following figure illustrates this transformation.

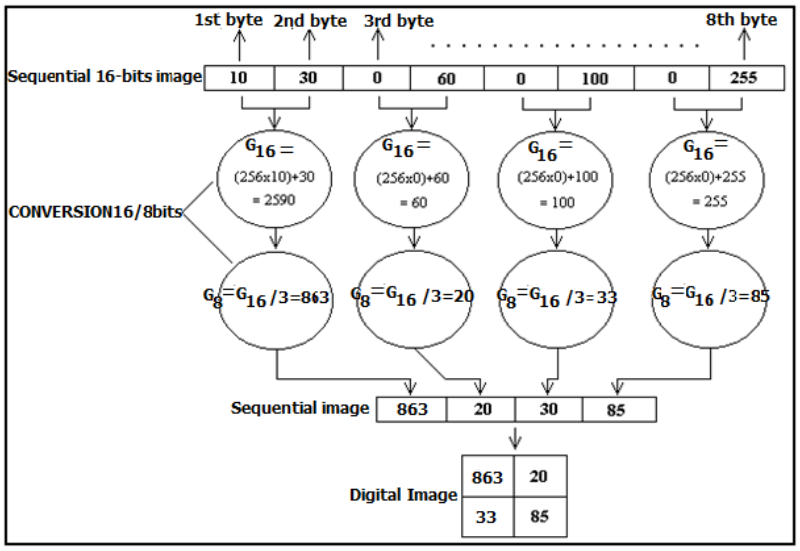


Figure 2.3. Example of transformation.

Since each pixel is coded on two bytes, B_u and B_l, the number of bytes per line is twice the number of pixels. The related algorithm is the following.

Algorithm 2.9 Conversion16_8bits_Intensities
Input:
NL, NC: integers; //Respectively the number of lines and columns of the image;
FileName: String; //Name of the 16-bits image file (file of bytes)
Local Variables:
x,y: integers ; // Coordinates of a pixel in the image
LB: Array[1,--,2NC] of bytes; // Line of bytes
IMA: Array[0,--,NL-1;0,--,NC-1] of reals; // to store temporarily the matrix image
Output
IM8: Array[0,--,NL-1; 0,--,NC-1] of bytes; // Converted image
BEGIN
Open the image file named FileName;
FOR y varying from 0 to **NL**-1 **DO**
BEGIN
Read 2NC bytes in the image file and store in LB;
FOR x varying from 0 to **NC**-1 **DO**
IMA[y,x] ← (**LB**[2*x]*256+LB[2*x+1])/3;
END
Calculate the statistics (AVG, AvgSq and SD) on data of the matrix IMA;
FOR y varying from 0 to NL-1, **DO**
FOR x varying from 0 to NC-1, **DO**
IM8[y,x] ← **minimum{ENT(**(255*IMA[y,x])/(**AVG** + 3***SD**)**), 255}**;
Close the image file *FileName*;
END

2.5 Exercises

- Present the principle of image acquisition.
- How are images data stored on a computer data support?

- Describe the various methods of reading and displaying digital images.
- The figure 2.4 below represents a 16-bits image
    - What are the number of lines and columns of this image, knowing that the image has more than one line?
    - Convert this image into an 8-bits image, considering that it is a 16-bits in amplitude.
    - Convert this image into an 8-bits image, considering that it is a 16-bits in intensities.
    - Identify on the 16-bits image the bytes of the ZOI starting from (1,1) and ending on (3,3).
- Write a program that reads, from a magnetic band (file of bytes) and displays on a screen, an 8-bits image, using any programming language of your choice.
- Write a program that converts a 16-bits image in amplitude into an 8-bits image, using any programing language of your choice;
- Write a program that converts a 16-bits image in intensities into an 8-bits image, using any programing language of your choice;
- Write a program that reads a zone of Interest (ZOI) from a 16-bits image and displays on a computer screen.

0	27	3	1	0	0	1	191	2	3	0	1	0	32	1	0
1	2	3	0	1	125	4	0	0	44	2	136	2	3	1	165

Figure 2.4. a 16-bits image.

CHAPITRE 3. IMAGE CONVOLUTION WITH A LINEAR MASK

3.1 Introduction

In the domain of signals and systems, the convolution is an operation that permits to obtain the output y of a system, given an input signal x and an impulse answer h of the system as illustrated below.

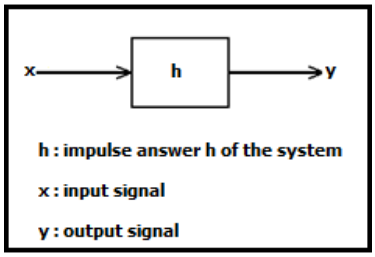


Figure 3.1. Illustration of a system including its input and output elements.

Given such a system, the output signal y is obtained by convoluting the input signal x with the impulse answer h of the system, according to the following equation.

$$y = x * h \Leftrightarrow y(t) = \int_0^t x(t-\lambda)h(\lambda)d\lambda \qquad \text{(Eq. 3.1)}$$

In case of digital signals such as Digital Image, this convolution operation is discrete. The process of convolution in this case is given by the following equation:

$$y(k,l) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} x(k-m,l-n)\cdot h(m,n) \qquad \text{(Eq. 3.2)}$$

In this expression, x represents an image function and h represents a discrete mask of convolution. M and N are respectively the number of lines and the number of columns of the image. Y is the result of the convolution of the image x with the mask of convolution h. In what follows, we present the detailed principle of an image convolution with a linear mask.

3.2 Digital image convolution principle

A linear mask is a matrix with entire coefficients. Following is an example of linear mask.

$$M = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

The main steps of the linear convolution are the following:

1) Center the mask on each pixel of the image;
2) Calculate the linear combination of the coefficients of the mask with the grayscales of the surrounding pixels of the central pixel of the image window (point on which the mask is centred);
3) Replace the grayscale of the central point by the result of the previous combination.
4) Normalise the obtained result (convert the grayscales of the convoluted image in the range of variation of grayscales of the original image).

For example, let consider the following image I and the previous mask M.

$$I = \begin{bmatrix} 4 & 0 & 100 & \dots & 240 \\ 2 & 7 & 4 & \dots & 100 \\ 6 & 3 & 1 & \dots & 50 \\ . & . & . & . & . \\ . & . & . & . & . \\ 190 & 0 & 255 & 75 & 40 \end{bmatrix}$$

Let consider the point of coordinates (1,1) as our central point. Its grayscale is 7. This point corresponds to the centre of the mask, the value 4. We can extract an image window W from the digital image I and centred on the pixel (1,1).

$$W = \begin{bmatrix} 4 & 0 & 100 \\ 2 & 7 & 4 \\ 6 & 3 & 1 \end{bmatrix}$$

The surrounding points of the central point of the image window correspond to the values -1 in the mask. The result of convolution is obtained by multiplying the two matrices (image window and mask) term per term and the result replaces the central value of the image window.

C = (-1 x 4)+(-1 x 0)+(-1 x 100)+(-1 x 2)+(4 x 7)+(-1 x 4)+(-1 x 6)+(-1 x 3)+(-1 x 1)

=-92.

At the end of the process, the results is normalised, using the following equation.

$$G_{new} = ENT\left(\frac{G_{old} \times C}{G_{\max}}\right). \qquad \text{(Eq. 3.3)}$$

G_{new} is the grayscale of the current pixel after normalisation. G_{old} is the maximum grayscale in the original image. $G_{\max}$ is the maximum grayscale in the convoluted image before normalisation and C is the value of the current convoluted pixel before normalisation and ENT is the entire part operator.

For example, let suppose that the maximum grayscale of the convoluted image before normalisation is 200 and that the maximum grayscale in the original image is 255. Then the normalised convoluted pixel (1,1) of the previous image I is given by:

$$G_{(1,1)} = ENT\left(\frac{200\times -92}{255}\right) = ENT(-72{,}15) = -73 \qquad \text{(Eq. 3.4)}$$

The normalisation is necessary to visualise accurately the convoluted image and to better appreciate the action of convolution on the image. The normalisation aims at bringing the grayscales of the convoluted image in the range of variation of grayscales of the original image so that one can better appreciate the changes operated by the convolution. Following is an algorithm that convolutes an image with a linear mask.

Algorithm 3.1 Linear_Convolution
Input:
NL, NC: integers; //Respectively the number of lines and columns of the image;
My, Mx: integers; //Respectively the number of lines and columns of the mask;
IMG: Array[0--NL-1; 0--NC-1] of bytes; // Original image
M: Array[0--My-1;0--Mx-1] of reals; // the linear mask;
Local Variables:
x,y: integers ; // Coordinates of a pixel in the image
Output:
IMA: Array[0,--,NL-1; 0,--,NC-1] of bytes; // Convoluted image
BEGIN
FOR y varying from 0 to **NL**-1 **DO**
FOR x varying from 0 to **NC-**1 **DO**
Convolution_Pixel(x, y, IMG, M, M_x, M_y, C);
IMA[y,x] ← C ;
END
//Normalisation
G_{max} ←Maximum value (grayscale) in IMA;
G_{old} ←Maximum grayscale in IMG;
FOR y varying from 0 to **NL**-1 **DO**
FOR x varying from 0 to **NC-**1 **DO**
IMA[y,x]← (**IMA[y,x]*** G_{old})/G_{old};
END

Convolution_Pixel is a sub-program that convolutes a single pixel (x,y) in an image IMG, with a linear mask M of size Mx columns and My lines. The result is stored in the variable C. Below is the specification of this sub-algorithm.

Algorithm 3.2 Convolution_Pixel
Input:
X_0, Y_0: integers; //coordinates of the pixel to convolute
M_x, M_y: integers; //resp. number of columns and number of lines of the mask;
IMG: Array[0--NL-1; 0--NC-1] of bytes; // Original image
M: Array[0--My-1;0--Mx-1] of reals; // The linear mask

```
Local Variables:
        x,y: integers; // Coordinates of a pixel in the image
        I,J; Integers;
Output
       C: integer; // Convoluted pixel
BEGIN
    C ← 0;
    FOR y varying from Y0-Wy/2 to Y0+Wy/2 DO
        FOR y varying from X0-Wx/2 to X0+Wx/2 DO
            BEGIN
                I ← x – Xo + (L/2);
                J ← y – Yo + (H/2);
                C ← C + (IMG[y,x] * M[J, I]);
        END
END
```

3.3 Case of image border pixels

3.3.1. Principle

The principle of image convolution cannot be applied on every point of the image, notably the image border points. To make it possible, we have to add some virtual columns and lines to the image so that, all the border points become internal points of the filled image. The number of lines and columns depends on the size of the mask of convolution. Considering a mask of size Mx columns and My lines, we can consider the point **($M_y/2$,$M_x/2$)** as the centre of the mask. In this case, we have to add **ENT($M_x/2$)** empty columns before the first column and **ENT(Mx-1-($M_x/2$))** empty columns after the last column of the image. **ENT** being the entire part function. Similarly, we must add **ENT($M_y/2$)** empty lines before the first line and **ENT(Mx-1-($M_y/2$))** empty lines after the last line of the image.

For example, with a mask of size 3 columns and 4 lines, the centre of the mask is the pixel at coordinates (2,1). In this case, one must add 1 column before the first column, 1 column after the last column, 2 lines before the first line and 1 line after the last line of the image. Now that we know the number of empty columns and lines to add, the problem is how fill them. For this purpose, there are various techniques of filling these empty points. But in this chapter, we will present only three of them, notably the Zeros method, the symmetric method and the circular symmetric method.

3.3.2 Zero's method

In this approach, one consider that any point that doesn't belong to the image has the grayscale 0. This method consists in filling all the added empty points with the value 0 before doing the convolution. The following figure illustrates the border filling process with Zeros method, with a linear mask of size 3x3.

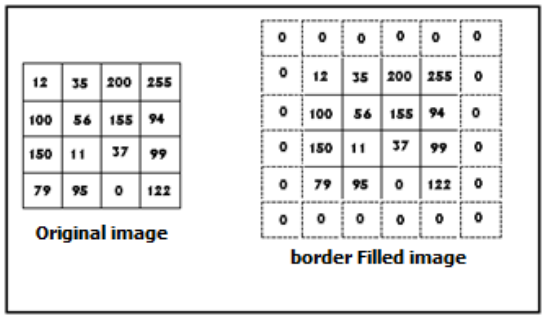


Figure 3.2. Example of border filling with zeros method.

3.3.3 Symmetric method

In this method, the first (resp. the last) empty lines are filled by considering the corresponding symmetric lines in the image, with respect to the first (resp. the last) line of the image. Similarly, the first (resp. the last) empty columns are filled by considering the corresponding symmetric columns in the image, with respect to the first (resp. the last) column of the image. In the same conditions as previously, the following figure illustrates this method.

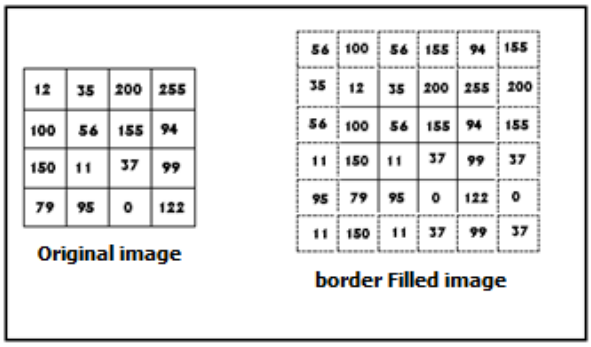


Figure 3.3. Example of border filling with symmetric method.

3.3.4 Circular symmetric method

In this method, one considers that the last column is followed by the first column and the last line followed by the first line. With the same conditions as previously, the following figure illustrates an example of circular symmetric method for border filling.

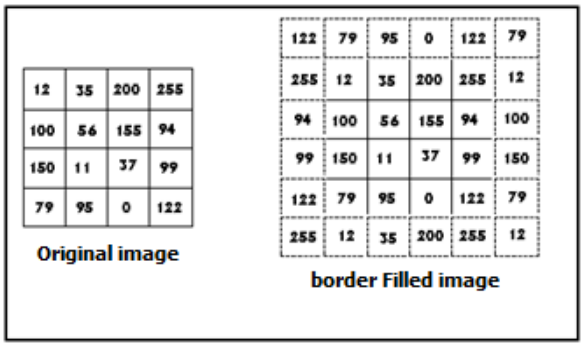


Figure 3.4. Example of border filling with circular symmetric method.

3.4 Exercises

- Describe the principle of image convolution.
- What is the usefulness of border filling in image convolution?
- Present the principle of the various methods of border filling.
- Write a program in any programming language of your choice that convolutes a digital image with a linear mask, using successively the three methods of border filling.
- Realise manually the convolution of the digital image defined in the following figure, using the given mask and successively the three methods of border filling.

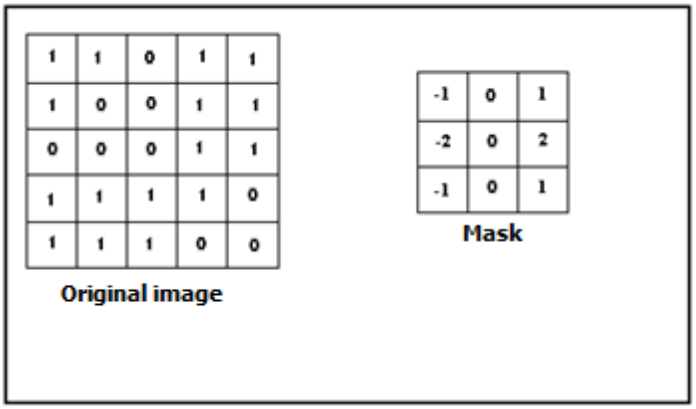


Figure 3.5. Digital image and the mask of convolution

CHAPTER 4 THE FILTERING OF IMAGES

4.1 Introduction

Satellite images often contain noise. This noise is due either to the surrounding nature, or to the nature of the sensors embarked on the satellites. Such images often have a blurred aspect. It is then necessary to filter them before any usage. Filters are classified in two classes: linear filters and non-linear filters. SAR Images often contain a particular type of noise called glistening. Only non-linear filters are suitable for the filtering of such images.

4.2 Linear filtering

Linear filtering consists in applying linear masks of convolution to the image. A linear mask of classic filtering is for example the average mask M defined by: $M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

Filtering an image with the average filter consists in convoluting the image with the average mask. The following image presents an example of SAR image filtered with the average filter.

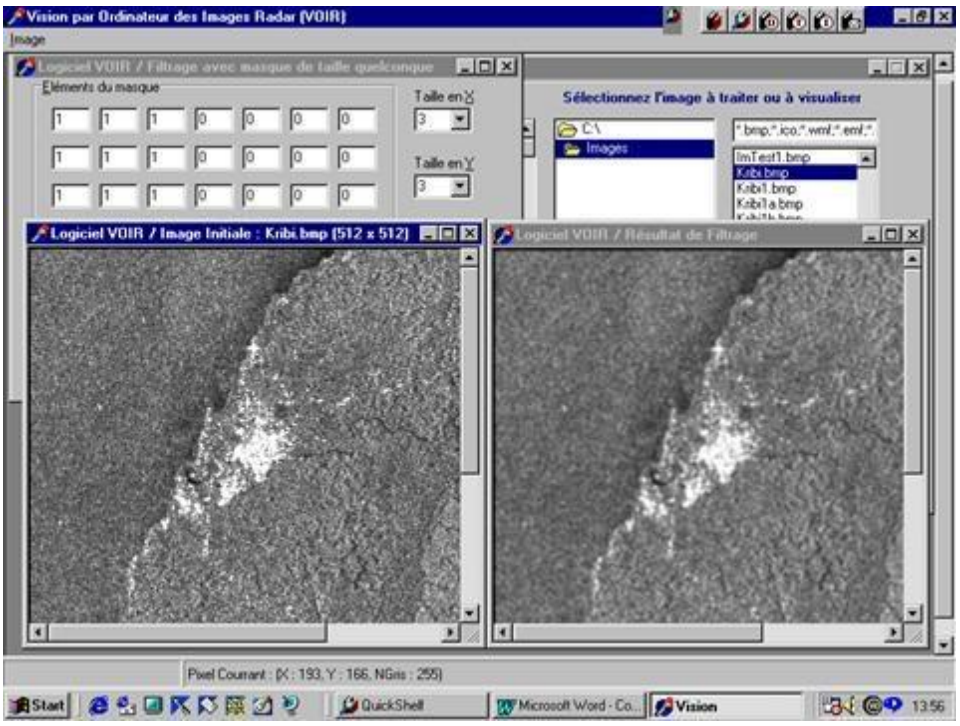


Figure 4.1. Example of linear filtering (average filter) realised on an ERS1 image of Kribi.

Linear masks are defined in two or four directions.

4.2.1 Filters defined in 2-directions

This section presents some examples of linear filters defined in two directions.

Masks of **Robert**:

Horizontal direction	$M_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$	Vertical direction	$M_y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

Masks of **Sobel**

Horizontal direction	$M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$	Vertical direction	$M_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

Masks of **Prewitt**

Horizontal direction	$M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$	Vertical direction	$M_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Masks of **Kirsh**

Horizontal direction	$M_x = \begin{bmatrix} -3 & -3 & 5 \\ -3 & 0 & 5 \\ -3 & -3 & 5 \end{bmatrix}$	Vertical direction	$M_y = \begin{bmatrix} -3 & -3 & -3 \\ -3 & 0 & -3 \\ 5 & 5 & 5 \end{bmatrix}$

4.2.2 Filters defined in 4-directions

The previous filters are also defined in 4-directions, notably the 0° direction corresponding to the horizontal direction, the 90° direction corresponding to the vertical direction, the 45° direction corresponding to the first diagonal direction and the 135° direction corresponding to the second diagonal direction. Below are the specifications of these filters.

Masks of **Robert**:			
0° direction	$M_{0^\circ} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$	90° direction	$M_{90^\circ} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$
45° direction	$M_{45^\circ} = \begin{bmatrix} -2 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$	135° direction	$M_{135^\circ} = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$
Masks of **Sobel**			
Horizontal direction	$M_{0^\circ} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$	Vertical direction	$M_{90^\circ} = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$
45° direction	$M_{45^\circ} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$	135° direction	$M_{135^\circ} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

Masks of **Prewitt**			
Horizontal direction	$M_{0°} = \begin{bmatrix} -1 & 1 & 1 \\ -1 & -2 & 1 \\ -1 & 1 & 1 \end{bmatrix}$	Vertical direction	$M_{90°} = \begin{bmatrix} -1 & -1 & -1 \\ 1 & -2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
45° direction	$M_{45°} = \begin{bmatrix} -1 & -1 & 1 \\ -1 & -2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	135° direction	$M_{135°} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & 1 \\ -1 & -1 & 1 \end{bmatrix}$
Masks of **Kirsh**			
Horizontal direction	$M_{0°} = \begin{bmatrix} -3 & -3 & 5 \\ -3 & 0 & 5 \\ -3 & -3 & 5 \end{bmatrix}$	Vertical direction	$M_{90°} = \begin{bmatrix} -3 & -3 & -3 \\ -3 & 0 & -3 \\ 5 & 5 & 5 \end{bmatrix}$
45° direction	$M_{45°} = \begin{bmatrix} -3 & -3 & -3 \\ -3 & 1 & 5 \\ -3 & 5 & 5 \end{bmatrix}$	135° direction	$M_{135°} = \begin{bmatrix} -3 & 5 & 5 \\ -3 & 0 & 5 \\ -3 & -3 & -3 \end{bmatrix}$

4.2.3 Application

Let consider for example the image below acquired by ERS-1 on the Cameroonian Atlantic coast.

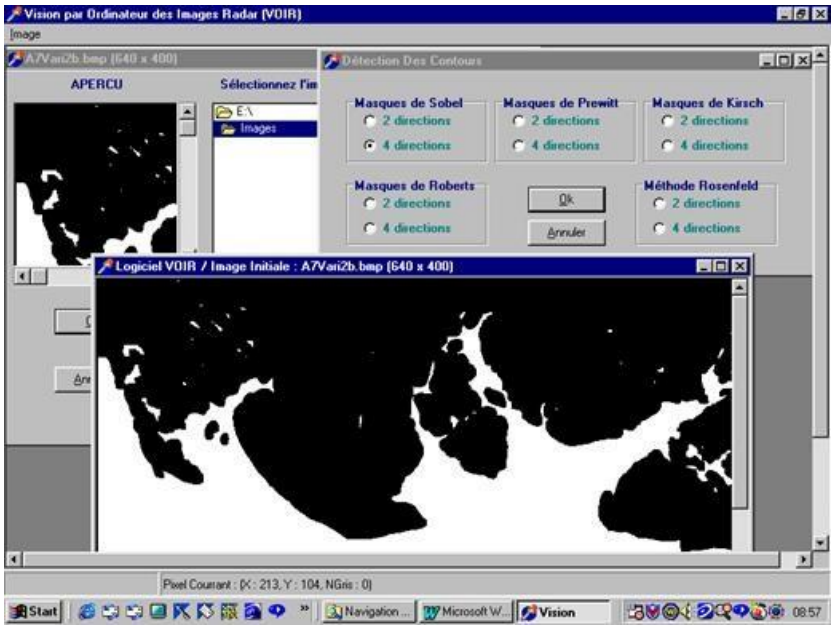


Figure 4.2 Experimental image (ERS1 Cameroonian Atlantic coast image).

The image below presents the result of shoreline detection by filtering of the experimental image with the Sobel's directional filter. This result is obtained by convoluting the image with each of the 4-directional masks of Sobel, notably 0°, 45°, 90° and 135° directions. Each pixel of the experimental image is then convoluted 4 times. Finally, the value of the filtered pixel is the maximum value of the 4 convolutions of the pixel. This value is considered as the amplitude of the gradient of the considered pixel.

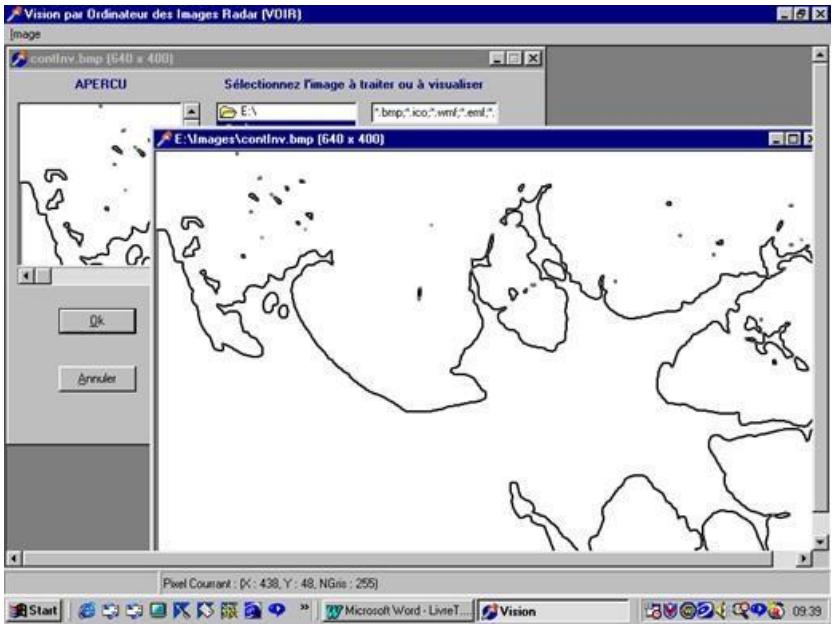


Figure 4.3 Result of the shoreline detection by Sobel's directional masks.

4.3 Filtering of the glistening on SAR images

4.3.1 Preliminary

SAR images often contain a particular type of noise called **glistening**. This noise is due to the distribution of a coherent signal in an atmosphere characterised by unpredictable variations. Glistening complicates interpretation and processing of radar images. Unfortunately, linear filters are not able to face this problem. This failure of linear filters such as average or median filters is due to the fact that glistening is a multiplicative noise. This is why other categories of filters named **non-linear** or **adaptive** filters were developed for the reduction or elimination of such noise. In the following sections, we study some of them. These new techniques model the image signal as a product of the real signal with the noise as specified in the following equation.

$$z(k,l)=y(k,l)*u(k,l) \qquad \text{(Eq. 4.1)}$$

Where z(k,l) is the obtained value (intensity or amplitude) of the pixel at location (k,l) of the SAR image; y(k,l) is the real radiometric value of the pixel and u(k,l) is the noise that affected the pixel. This noise is characterised by a distribution of average 1 and a Standard Deviation σ_u. Thus, there is a linear relation between the Average and the Standard Deviation in homogenous regions of the image. This situation helps to test the hypothesis of a multiplicative noise and to confirm the homogeneity of a region in the image.

4.3.2 local statistics

Since it is not possible to obtain an accurate model of the original signal, the image itself can be used to estimate some required statistics, notably the Average and the Variance *a priori* of the signal, based on the local Average and the local variance in an image window of size n x n. From the multiplicative model of the noise, the following estimates of the Average and the Variance *a priori* are obtained.

$$\bar{y} = \frac{\bar{z}}{\bar{u}} = \bar{z} \qquad and \qquad s_y^{\,2} = \frac{s_z^{\,2} - \bar{z}^2 s_u^{\,2}}{s_u^{\,2} + 1} \qquad \text{(Eq. 4.2)}$$

We suppose that we have a linear filter expressed in the form where is the estimation of the minimum of the squawroot of the signal y, a and b being chosen such as to minimize the error in the least squares sense. It has been demonstrated that the best estimate of y is given by the following equation.

$$\tilde{y} = \bar{z} + b(z - \bar{z}) \qquad with \quad b = \frac{s_y^{\,2}}{s_z^{\,2}}, \qquad \bar{y} = \bar{z} \qquad \text{(Eq. 4.3)}$$

The unique input parameter of the filter is which depends on the SAR multi-view treatment. It is necessary that should be always positive. Otherwise, it must be set to zero. This problem can be escaped if a lower limit of regions homogeneity is conveniently chosen. In the Lee original method, a linear model is introduced, with the optimal value of *b* expressed in the following equation.

(Eq. 4.4)

By substitution, the following value of *b* is obtained:

(Eq. 4.5)

Knowing that , this linearisation doesn't affect the filtering of glistening in the case of a 4-views image in amplitude. But in the case of a 1-view SAR image, the exact expression must be used. The estimation of the standard deviation of the noise is a function of the number of views and a quality factor of the SAR image. Below is the algorithm that estimates the Standard Deviation of the glistening.

4.3.3 Glistening Standard Deviation estimation

The estimation of the Standard Deviation of the noise is based on the Gamma function given by the following algorithm.

Algorithm 4.1 Gamma_Function_Estimation: **real**;

-*Input:* N, integer representing the number of views of the image;

-*Local variables:* **a, t:** reals, **k:** integer;

BEGIN

a ← 1.7724538509;
t ← 1;
FOR k varying from 1 to N **DO**
t ← t*(2k–1);
Return (a*t)/2^N;
END

With the Gamma function estimated, it is now possible to estimate the Standard Deviation of the noise. The related algorithm is the following.

Algorithm 4.2. Noise_Standard_Deviation_Estimation: Real;

-*Input:* N: **integer** representing the number of views of the image;

Q: **integer** representing the factor or type of the image (Q=0 for 16-bits images in intensities; Q=1 for 8-bits images in amplitude of type squawroot of intensities; Q=2 for 8-bits n amplitudes of type average of amplitudes);

BEGIN

IF(Q = 0) **THEN** Noise_Standard_Deviation ← $1/\sqrt{N}$;

ELSEIF(Q = 1) **THEN** Noise_Standard_Deviation ← $\sqrt{0.273/N}$;

ELSEIF(Q = 2) **THEN** Noise_Standard_Deviation ← $\sqrt{N \times \left(\frac{(N-1)!}{Gamma(N)}\right)^2 - 1}$

END

Following is the algorithm of Lee for the filtering of glistening on SAR images.

4.3.4 LEE Filter of local statistics

The following algorithm describes the process of filtering a SAR image affected by glistening noise, using Lee filter.

Algorithm 4.3 Lee_Glistening_Filter

Input:

IMG: Original image;

NL, NC: **integer** representing respectively the number of lines and the number of columns of the image IMG;

n: **integer** representing the size of the image window to consider;

Output:

IMA: Filtered image;

Local variables:

Ima$_f$: **Array[1..NL ; 1..NC]** of **reals;**

i, j: **integers;**

a, b: **reals;**

BEGIN

1. Initialise the intermediary image table Ima$_f$ with the original image IMG;
2. **FOR** j varying from 0 to NL-1 **DO**

   FOR i varying from 0 to NC-1 **DO**

   BEGIN

   a) Extract, from Ima$_f$, an image window W centred on the pixel (i,j) ;

   b) Calculate the local average $\bar{z} = \sum_{a=0}^{n-1}\sum_{b=0}^{n-1} W[a,b]/n^2$ of W;

   c) Calculate the local variance $S_z^{\,2} = (1/(n^2-1)) \times \sum_{a=0}^{n-1}\sum_{b=0}^{n-1}\left(ima_f[a,b]-\bar{z}\right)^2$ of W;

   d) Estimate the Standard Deviation of the noise in the image window:

$$\begin{cases} S_u \;<-\; Noise\_Standard\_Deviation(N,Q) \\ N: Number\ of\ views\ of\ the\ image \\ Q: Image\ Factor\ representing\ the\ type\ of\ the\ image \end{cases}$$

   e) Estimate the Average end the Variance a priori of the same image window:

$$\begin{cases} \bar{y} = \bar{z} \\ S_y^{\,2} = \left(S_z^{\,2} - S_u^{\,2}\right)/\left(S_u^{\,2}+1\right) \end{cases}$$

   f) Verify the sign of the variance a priori:

   If $S_y^{\,2} \le 0$ *Then* $S_y^{\,2} = 0$

   g) Calculate the coefficient *b*:

$$b = \frac{S_y^{\,2}}{\bar{z}^2 \cdot S_u^{\,2} + S_y^{\,2}}$$

   h) Change the grayscale Ima$_f$[i,j] by the following assignment:

   $Ima_f[i,j] \leftarrow \bar{z} + b\left(Ima_f[i,j] - \bar{z}\right)$;

   END

3. Transform the real values of Ima_f into integers and assign to the image table IMA:

FOR j varying from 0 to NL-1 **DO**

FOR i varying from 0 to NC-1 **DO**

$IMA[i, j] \leftarrow ENT(\mathrm{Im}a_f[i, j])$; *ENT* is the entire part function.

END

4.3.5 Alternated Lee filter

The alternated Lee filter is a variant of the Lee filter previously presented. Its algorithm is the following:

Algorithm 4.4 Alternated_Lee_Glistening_Filter

Input:

IMG: Original image;

NL, NC: **integer** representing respectively the number of lines and the number of columns of the image IMG;

n: **integer** representing the size of the image window to consider;

Output:

IMA: Filtered image;

Local variables:

Ima_f: **Array[1..NL ; 1..NC]** of **reals**;

i, j: **integers**;

a, b: **reals**;

BEGIN

1. Initialise the intermediary image table Ima_f with the original image IMG;
2. **FOR** j varying from 0 to NL-1 **DO**

FOR i varying from 0 to NC-1 **DO**

BEGIN

a) Extract, from Ima_f, an image window W centred on the pixel (i,j) ;

b) Calculate the local average $\bar{z} = \sum_{a=0}^{n-1}\sum_{b=0}^{n-1} W[a,b]/n^2$ of W;

c) Calculate the local variance $S_z^{\,2} = (1/(n^2-1)) \times \sum_{a=0}^{n-1}\sum_{b=0}^{n-1}(\mathrm{Im}a_f[a,b] - \bar{z})^2$ of W

d) Estimate the Standard Deviation of the noise in the image window W:

$$\begin{cases} S_u \;<-\; Noise\_Standard\_Deviation(N,Q) \\ N: Number\ of\ views\ of\ the\ image \\ Q: Image\ Factor\ representing\ the\ type\ of\ the\ image \end{cases}$$

e) Estimate the Standard Deviation *a priori* of the same image window:

$$\begin{cases} If \quad (\bar{z} \neq 0) \quad Then \quad S_y \leftarrow \sqrt{S_z^{\,2}} \,/\, \bar{z} \\ Else \quad S_y \leftarrow 1 \end{cases}$$

f) Compare the Standard deviation *a priori* with the Standard Deviation of the noise and deduce the grayscale of the filtered pixel:

If ($S_y < S_u$) **Then**

$$\text{ima[i, j]} \leftarrow \text{ENT}(\bar{z})$$

ELSE

BEGIN

$$a \leftarrow S_y^{\,2} + S_u^{\,4}$$

$$b \leftarrow \text{Im}\, a_f[j,i] \times \left(S_y^{\,2} - S_u^{\,2}\right) + \bar{z} \times \left(S_u^{\,2} + S_u^{\,4}\right)$$

$$b \leftarrow b/a$$

$$IMA[j,i] \leftarrow ENT(b)$$

END

END

4.3.6 Maximum of Probability *a Posteriori* (MAP) filter

This adaptive filter is based on the maximisation of the probability *a posteriori* (MAP), p(y/z) of the signal y(k,l), knowing z(k,l): . Given an N-look image in intensities, P(y/z) follows thesquare:

In contrary to the Lee filter that doesn't require any model for the signal, the MAP filter supposes that the signal follows a Gaussian distribution: and being respectively the Average and the Variance estim!€ated through local statistics in an image window. The related algorithm is the following:

Algorithm 4.5 MAP_Glistening_Filter

Input:

IMG: Original image;

NL, NC: **integer** representing respectively the number of lines and the number of columns of the image IMG;

n: **integer** representing the size of the image window to consider;

Output:

IMA: Filtered image;

Local variables:

Ima_f: **Array[1..NL ; 1..NC]** of **reals**;

i, j: **integers**;

a, b: **reals**;

BEGIN

1. Initialise the intermediary image table Ima_f with the original image IMG;
2. **FOR** j varying from 0 to NL-1 **DO**

   FOR i varying from 0 to NC-1 **DO**

   BEGIN

   a) Extract, from Ima_f, an image window W centred on the pixel (i,j) ;

   b) Calculate the local average $\bar{z} = \sum_{a=0}^{n-1}\sum_{b=0}^{n-1} W[a,b]/n^2$ of W;

   c) Calculate the local variance $S_z^{\,2} = (1/(n^2-1)) \times \sum_{a=0}^{n-1}\sum_{b=0}^{n-1} (W[a,b]-\bar{z})^2$ of W;

   d) Estimate the Standard Deviation of the noise in the image window W:

$$\begin{cases} S_u \;<-\; Noise\_Standard\_Deviation(N,Q) \\ N: Number\ of\ views\ of\ the\ image \\ Q: Image\ Factor\ representing\ the\ type\ of\ the\ image \end{cases}$$

   e) Estimate the Standard Deviation *a priori* of the same image window:

$$\begin{cases} If \quad (\bar{z} \neq 0) \quad Then \quad S_y \leftarrow \sqrt{S_z^{\,2}}/\bar{z} \\ Else \quad S_y \leftarrow 0 \end{cases}$$

   f) Compare the Standard deviation *a priori* with the Standard Deviation of the noise and deduce the grayscale of the filtered pixel:

If ($S_y < S_u$) **Then**

$ima[i, j] \leftarrow ENT(\bar{z})$

ELSE

BEGIN

$a \leftarrow S_y^2 \times (1 + S_u^2)$

$b \leftarrow \mathrm{Im}\, a_f[j,i] \times (S_y^2 - S_u^2) + \bar{z} \times S_u^2 \times (1 + S_y^2)$

$IMA[j,i] \leftarrow ENT(b/a)$

END

END

4.3.7 Gamma filter

This filter is similar to the previous one, but takes into consideration the number of views of the image to filter. Its algorithm is the following.

Algorithm 4.6 Gamma_Glistening_Filter

Input:

IMG: Original image;

NL, NC: **integer** representing respectively the number of lines and the number of columns of the image IMG;

n: **integer** representing the size of the image window to consider;

N_{views}: Number of the views of the image to filter;

Output:

IMA: Filtered image;

Local variables:

Ima_f: **Array[1..NL ; 1..NC]** of **reals**;

i, j: **integers**;

a, b, t_1, t_2, t: **reals**;

BEGIN

1. Initialise the intermediary image table Ima_f with the original image IMG;
2. **FOR** j varying from 0 to NL-1 **DO**

   FOR i varying from 0 to NC-1 **DO**

   BEGIN

   a) Extract, from Ima_f, an image window W centred on the pixel (i,j) ;

b) Calculate the local average $\bar{z} = \sum_{a=0}^{n-1}\sum_{b=0}^{n-1} W[a,b]/n^2$ of W;

c) Calculate the local variance $S_z^{\,2} = (1/(n^2-1)) \times \sum_{a=0}^{n-1}\sum_{b=0}^{n-1} (W[a,b]-\bar{z})^2$ of W;

d) Estimate the Standard Deviation of the noise in the image window W:

$$\begin{cases} S_u \;<-\; Noise\_Standard\_Deviation(N,Q) \\ N: Number\ of\ views\ of\ the\ image \\ Q: Image\ Factor\ representing\ the\ type\ of\ the\ image \end{cases}$$

e) Estimate the Standard Deviation *a priori* of the same image window:

$$\begin{cases} If \quad (\bar{z} \neq 0) \quad Then \quad S_y \leftarrow \sqrt{S_z^{\,2}}/\bar{z} \\ Else \quad S_y \leftarrow 0 \end{cases}$$

f) Compare the Standard deviation *a priori* with the Standard Deviation of the noise and deduce the grayscale of the filtered pixel:

If (S_y^2 - S_u^2) $\neq 0$ **Then**

BEGIN

$\alpha \leftarrow (1+S_u^{\,2})/(S_y^{\,2}-S_u^{\,2})$

$t_1 \leftarrow \bar{z}\cdot(\alpha - N_{views} - 1)$

$t_2 \leftarrow t_1^{\,2} + (4\alpha \cdot N_{views} \cdot IMA_f[i,j]\cdot \bar{z})$

$t \leftarrow t_1 + \sqrt{t_2}$

$IMA[i,j] \leftarrow ENT[t/(2\alpha)]$

END

END

4.3.8 Frost filter

This adaptive filter supposes that the useful information has an exponential auto-correlation function. i.e. the image is stationary in a significant neighbourhood of the filtered pixel. In this case, the best estimate of the signal is: $\tilde{y} = \sum_{k=0}^{n-1}\sum_{l=0}^{n-1} k_1 a\, e^{[-a(|k-k_0|+|l-l_0|)]} z(k,l)$ where represents the coordinates of the central pixel of the image window of size n x n, is the grayscale of the pixel located at (k,l), k_1 is a normalisation constant chosen to set the sum of filter weights to one. The constant a is

given by: , where is the normalisation constant, chosen to insure that . The Forst algorithm specification is the following:

Algorithm 4.7 Frost_Glistening_Filter

Input:

IMG: Original image;

NL, NC: **integer** representing respectively the number of lines and the number of columns of the image IMG;

n: **integer** representing the size of the image window to consider;

N_{views}: Number of the views of the image to filter;

Output:

IMA: Filtered image;

Local variables:

IMA_f: **Array[1..NL ; 1..NC]** of **reals**;

i, j: **integers**;

a, b, t_1, t_2, t: **reals**;

BEGIN

1. Initialise the intermediary image table Ima_f with the original image IMG;
2. **FOR** j varying from 0 to NL-1 **DO**

   FOR i varying from 0 to NC-1 **DO**

   BEGIN

   g) Extract, from IMA_f, an image window W centred on the pixel (i,j) ;

   h) Calculate the local average $\bar{z} = \sum_{a=0}^{n-1}\sum_{b=0}^{n-1} W[a,b]/n^2$ of W;

   i) Calculate the local variance $S_z^{\,2} = (1/(n^2-1)) \times \sum_{a=0}^{n-1}\sum_{b=0}^{n-1} (W[a,b]-\bar{z})^2$ of W;

   j) Estimate the Standard Deviation of the noise in the image window W:

$$\begin{cases} S_u \;<-\; Noise\_Standard\_Deviation(N,Q) \\ N: Number\ of\ views\ of\ the\ image \\ Q: Image\ Factor\ representing\ the\ type\ of\ the\ image \end{cases}$$

   k) Calculate the following coefficient k: $k = 4/(n \cdot S_u^{\,2})$;

   l) Calculate the following constant a: $a = k \cdot S_z^{\,2} / \bar{z}^2$;

   m) Execute the following assignment:

$$IMA_f[i,j] = \sum_{m=0}^{n-1}\sum_{l=0}^{n-1} W[m,l]e^{X}; \quad with \quad X = -a(|m-i|+|l-j|)$$

END

3. Determine the maximum value IMA_{max} in IMA_f;
4. **FOR** j varying from 0 to NL-1 **DO**

   FOR i varying from 0 to NC-1 **DO**

$$IMA[j,i] \leftarrow ENT\left(255\frac{IMA_f}{IMA_{max}}\right)$$

END

4.4 Application

Let consider the following image of 3 lines and 3 columns (NL=3 and NC=3). Let filter this image with the Lee filter, using an image window of size 3x3 and the symmetry principle of border filling.

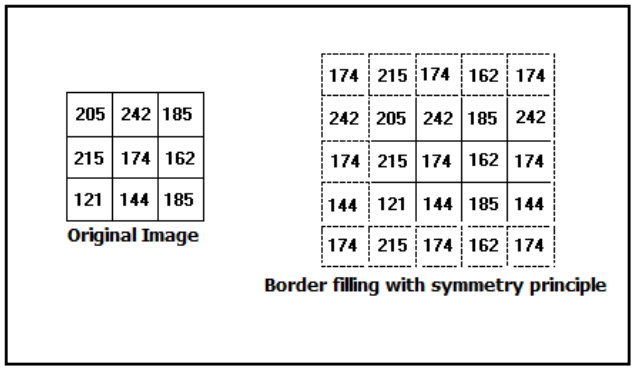


Figure 4.4 an example of digital image to filter.

The first pixel of this image is the pixel of coordinates (0,0), with grayscale 205. The image window centred on this pixel, considering the symmetry border filling principle is the following:

$$W_{205} = \begin{bmatrix} 174 & 215 & 174 \\ 242 & 205 & 242 \\ 174 & 215 & 174 \end{bmatrix}$$

1- The local average of this image window is calculated as follows:

$$\bar{z} = (174+215+174+242+205+242+174+215+174)/(3\times 3) = 201.66$$

2- The local variance of W_{205} is calculated as follows:

$$S_z^2 = ((174-201.66)^2 + (215-201.66)^2 + (174-201.66)^2 + (242-201.66)^2 + (205-201.66)^2 + (242-201.66)^2 + (174-201.66)^2 + (215-201.66)^2 + (174-201.66)^2) / (9-1) = 6682/8 = 835.25$$

3- Supposing that the image factor is Q=1 and that the number of views is 1, the Standard Deviation of noise is calculated as follows: $S_u \leftarrow \sqrt{0.273/1} = 0.5225$

4- The average and the variance *a priori* are given by:

$$\begin{cases} \bar{y} = \bar{z} = 201.66 \\ S_y^2 = \left(S_z^2 - S_u^2\right) / \left(S_u^2 + 1\right) = (835.25 - 0.5225^2) / (0.5225^2 + 1) = 655.9 \end{cases}$$

5- We notice that the variance *a priori* S_y^2 is positive, so nothing to change.

6- Let calculate the coefficient b:

$$b = \frac{S_y^2}{\bar{z}^2 \cdot S_u^2 + S_y^2} = \frac{655.9}{201.66^2 \times 0.5225^2 + 655.9} = 0.05578$$

7- The value of $IMA_f[0,0]$ is then given by:

$$IMA_f[j,i] = \bar{z} + b\left(IMA_f[j,i] - \bar{z}\right) = 201.66 + 0.05578 \times (205 - 201.66) = 201.84$$

8- The entire part of 201.84 is 201. So the filtered value IMA[0,0] of IMG[0,0] is 201. Similarly to what precedes, image windows centred on other pixels of the image are presented in the following figure:

$$W_{242} = \begin{bmatrix} 215 & 174 & 162 \\ 205 & 242 & 185 \\ 215 & 174 & 162 \end{bmatrix} \quad W_{185} = \begin{bmatrix} 174 & 162 & 174 \\ 242 & 185 & 242 \\ 174 & 162 & 174 \end{bmatrix}$$

$$W_{174} = \begin{bmatrix} 205 & 242 & 185 \\ 215 & 174 & 162 \\ 121 & 144 & 185 \end{bmatrix} \quad W_{162} = \begin{bmatrix} 242 & 185 & 242 \\ 174 & 162 & 174 \\ 144 & 185 & 144 \end{bmatrix}$$

$$W_{144} = \begin{bmatrix} 215 & 174 & 162 \\ 121 & 144 & 185 \\ 215 & 174 & 162 \end{bmatrix} \quad W_{185} = \begin{bmatrix} 174 & 162 & 174 \\ 144 & 185 & 144 \\ 174 & 162 & 174 \end{bmatrix}$$

$$W_{215} = \begin{bmatrix} 242 & 205 & 242 \\ 174 & 215 & 174 \\ 144 & 121 & 144 \end{bmatrix} \quad W_{121} = \begin{bmatrix} 174 & 215 & 174 \\ 144 & 121 & 144 \\ 174 & 215 & 174 \end{bmatrix}$$

Figure 4.5 Different image windows centred on pixels of the original image of Figure 4.4.

Following the same process as previously for these image windows, the results of intermediary calculus are presented in the table below.

Table 4.1. Intermediary results obtained during the Lee filtering process.

Window	W_{242}	W_{185}	W_{174}	W_{162}	W_{144}	W_{185}	W_{215}	W_{121}
$\bar{z}$	192.66	187.66	181.44	183.55	172.44	165.88	184.55	170.55
S_z^2	777.5	997	1357.77	1326.53	932.3	202.10	1937	983
S_u	0.5225	0.5225	0.5225	0.5225	0.5225	0.5225	0.5225	0.5225
S_y^2	610.54	782.97	1066.4	1041.83	732	158.54	1521.4	771.97
b	0.05682	0.0753	0.1060	0.1017	0.0827	0.02067	0.1406	0.0886
$IMA_f[i, j]$	195.46	187.46	180.65	181.35	170.08	166.27	188.83	166.15
$IMA[i, j]$	195	187	180	181	170	166	188	166

The following figure presents the final result of the filtering with the Lee filter.

Figure 4.6 Result of the Lee filtering of the digital image of figure 4.4.

No method of glistening filtering is absolutely better than another. Nevertheless, a given method can be better than another for a particular application.

4.5. Exercise

- What is the difference between a linear filter and a non-linear filter? Is one of the two better than another one?
- Consider the original image of figure 4.4 and the circular symmetry method.
  - Filter this image with Sobel, Kirsh, Robert and Prewitt filters;
  - Filter this image with Gama, MAP and Alternated Lee filter, considering n=3, N=2 and Q=2.

CHAPTER 5. THE MATHEMATICAL MORPHOLOGY APPLIED TO IMAGES

5.1 Introduction

The mathematical morphology is a technique of image analysis based on the notion of sets. Mathematical morphology can be applied on any type of digital image, but in this chapter, will focus only on binary images. The morphological operations are based on a structuring element which is a simple geometrical object with a classical form such as: rectangle, circle, square, hexagon, octagon...

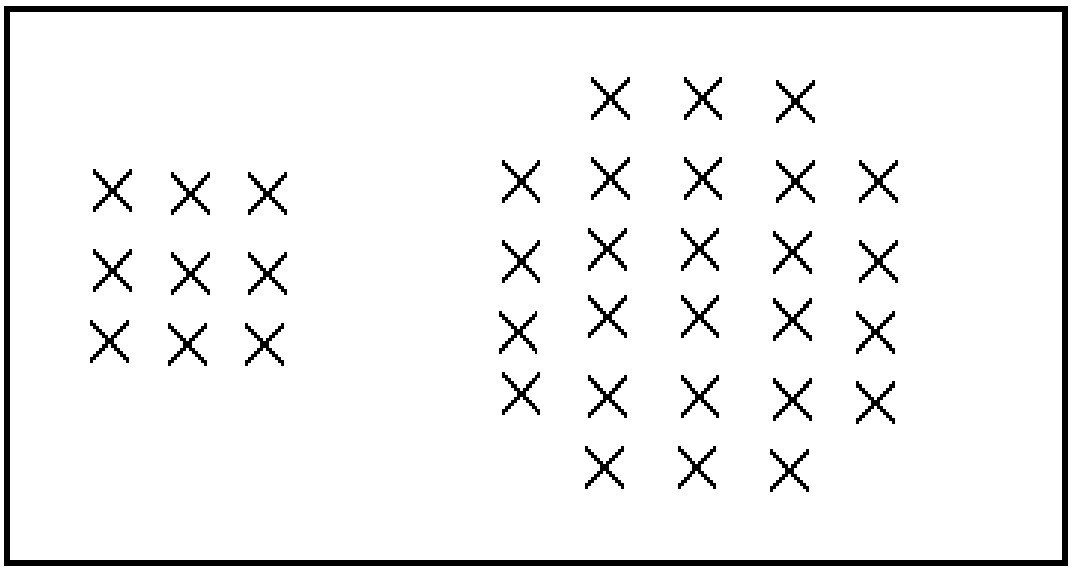


Figure 5.1. Square and octagonal structuring elements

For example $\boldsymbol{B} = \begin{bmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{1} \\ \mathbf{1} & \mathbf{-1} & \mathbf{1} \\ \mathbf{1} & \mathbf{-1} & \mathbf{1} \end{bmatrix}$ is a square structuring element to use, meaning that the middle column is disabled while the first and the third are enabled.

5.2 Image binarisation

The binarisation of an image consists in transforming the image into a 2-greyscales image called **binary image**. For this purpose, two binarisation thresholds T_1 and T_2 are usually considered. Below is a binarisation algorithme

5.2.1 Algorithm of image binarisation

Algorithm 5.1 Binarisation

Input: **IMG:** greyscales image;

NC, NL: Number of Columns and Number of lines respectively;

T_1, T_2: classification threshold.

Output: **IMA:** binarised image of NL lines and NC columns;

Local variable: **i,j:** integers;

BEGIN

FOR j varying from 0 to NL-1 **DO**

FOR i varying from 0 to NC-1 **DO**

IF (IMG[i, j]≥T_1) *and* (IMG[i, j]≤T_2) **THEN** IMA[i, j]←1;

ELSE IMA[i, j]← 0;

END

5.2.2 Example

Figure 5.2 shows an example of binarisation realised on a portion of ERS1 image acquired around the city of Douala in Cameroon.

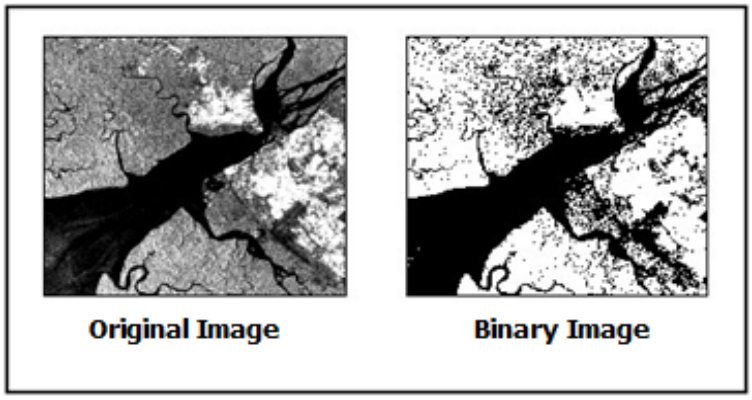


Figure 5.2. Binarisation of a 183x159 size portion of an ERS-1 image of the region of Douala with T_1=0 and T_2=100 for thresholds.

5.3 The erosion of a binary image

An image Erosion consists in reducing entities present in the image. For this purpose, a sort of image convolution is done by the mean of a structuring element. The structuring element is a sort of mask that enables some cells and disable others. The centre of the structuring element is each time placed on the current pixels and through enabled cells, greyscales of pixels are red for the erosion process of the current pixel identified by the centre of the mask. An image erosion can be repeated several times on the same image. In

this case, we talk of **erosion of size n, or of order n**, denoted by $E^{Bn}(X)$. The following algorithm specifies the related steps.

Algorithm 5.2 Image_Erosion
Input: IMG: binary image to erode;
$\boldsymbol{B} = \begin{bmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{1} \\ \mathbf{1} & \mathbf{-1} & \mathbf{1} \\ \mathbf{1} & \mathbf{-1} & \mathbf{1} \end{bmatrix}$ is the structuring element to use;
Output: IMA: eroded image;
Local variable: i,j: integers;
IMA_{b1}, IMA_{b2}: Array [0..NL-1;0..NC-1] of Booleans;

```
BEGIN
  FOR j varying from 0 to NL-1, DO
    FOR i varying from 0 to NC-1, DO
      BEGIN
        IMAb2[i,j]←FALSE ;
        IF (IMG[i,j]>0) THEN IMAb1[i,j]←TRUE;
        ELSE IMAb1[i,j]←FALSE ;
      END;
  Fill the borders of IMAb1 and IMAb2, using any of the three principles;
  FOR j varying from 0 to NL-1, DO
    FOR i varying from 0 to NC-1, DO
      BEGIN
        Extract an image window W from the boolean image IMAb1 centred on (j,i);
        IF (W[0, 0]) and (W[1,0]) and (W[2,0]) and (W[0,2]) and (W[1,2]) and (W[2,2]) THEN
          IMAb2[j,i] ← IMAb1[j,i];
        ELSE IMAb2[j,i] ← FALSE;
      END;
  FOR j varying from 0 to NL-1, DO
    FOR i varying from 0 to NC-1, DO
      BEGIN
        IF (IMAb2[j,i]) THEN
          IMA[j,i] ← 255;
        ELSE IMA[j,i] ← 0;
      END;
END
```

The following figure presents an example of a digital Image erosion.

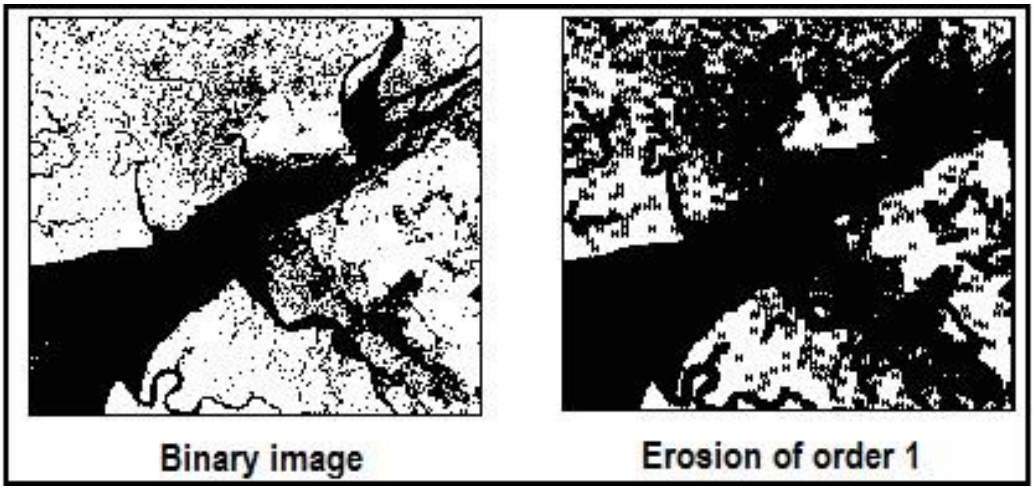


Figure 5.3. Erosion of a 183x159 size portion of SAR ERS1 image acquired on the region of Douala in Cameroon

5.4 The dilation of a binary image

In contrary to the erosion, Image dilation has the effect of increasing the size of entities. The process is similar to the one of image erosion. A dilation can also be repeated several times on the same image. In this case, we talk of **dilation of size n, or of order n, denoted by $D^{Bn}(X)$**. Following is the algorithm of digital image dilation.

Algorithm 5.3 Image_Dilation

Input: IMG: binary image to erode;
$\boldsymbol{B} = \begin{bmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{1} \\ \mathbf{1} & \mathbf{-1} & \mathbf{1} \\ \mathbf{1} & \mathbf{-1} & \mathbf{1} \end{bmatrix}$ is the structuring element to use;
Output: IMA: eroded image;
Local variable: i,j: integers;
IMA_{b1}, IMA_{b2}: Array [0..NL-1;0..NC-1] of Booleans;

```
BEGIN
  FOR j varying from 0 to NL-1, DO
    FOR i varying from 0 to NC-1, DO
      BEGIN
        IMAb2[i,j]←FALSE ;
        IF (IMG[i,j]>0) THEN IMAb1[i,j]←TRUE;
        ELSE IMAb1[i,j]←FALSE ;
      END;
  Fill the borders of IMAb1 and IMAb2, using any of the three principles;
```

```
  FOR j varying from 0 to NL-1, DO
    FOR i varying from 0 to NC-1, DO
        BEGIN
            Extract an image window W from the boolean image IMAb1 centred on (j,i);
            IF (W[0, 0]) OR (W[1,0]) OR (W[2,0]) OR (W[0,2]) OR (W[1,2]) OR (W[2,2]) THEN
               IMAb2[j,i] ← TRUE;
            ELSE IMAb2[j,i] ← IMAb1[j,i];
        END;
  FOR j varying from 0 to NL-1, DO
    FOR i varying from 0 to NC-1, DO
        BEGIN
            IF (IMAb2[j,i]) THEN
               IMA[j,i] ← 255;
            ELSE IMA[j,i] ← 0;
        END;
END
```

The following figure presents an example of image dilation.

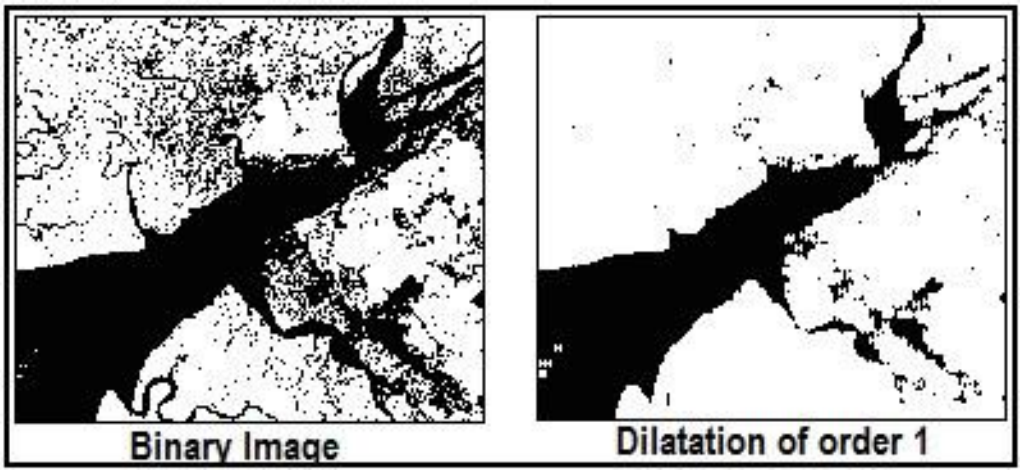


Figure 5.4. Dilation of a 183x159 size portion of SAR ERS1 image acquired on the region of Douala in Cameroon

5.5 The opening of a binary image

A binary image opening aims to eliminate narrow peninsulas of entities in the image by combining erosion and dilation. Denoted by $\mathbf{X_B}$, it consists in an erosion followed by a dilation. An **opening of size n**, denoted by $\mathbf{X_{Bn}}$ **can also be made** by making a series of n

erosions followed by a series of n dilations. The following image illustrates an opening of an ERS-1 SAR image acquired on the region of Douala in Cameroon.

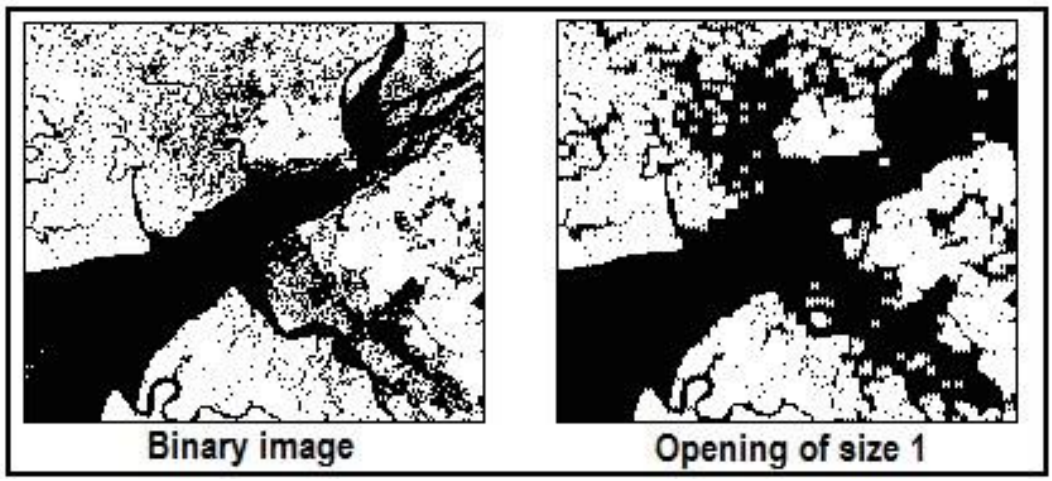


Figure 5.5. Opening of a 183x159 size portion of SAR ERS1 image acquired on the region of Douala in Cameroon

5.6 The closure of a binary image

In contrary to the opening of a binary image, the closure of a binary image is an operation that closes the bays and the small holes combining binary image erosion and dilation. The binary image closure is the reverse of the opening. It consists in first applying a dilation followed by an erosion. Obviously, a closure **of size n (or of order n)** denoted by $\mathbf{X^{Bn}}$ can be realised by applying a series of n dilations followed by a series of n erosions. The following figure presents an illustration of a binary image closure.

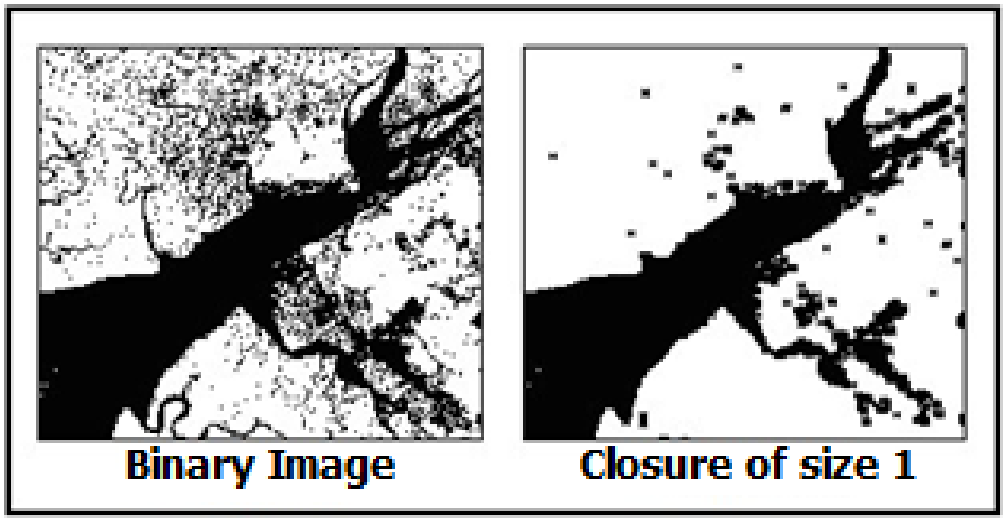


Figure 5.6. Closure of a portion of a SAR ERS-1 image of 183x159 size acquired on the region of Douala in Cameroon

5.7 The top hat form

Top Hat forms are filters used to extract small clear zones and small darken zones on an image. These filters are defined in two forms as follows:

1. The **White Top Hat (WTH)** is defined as the difference between a binary image and the closure of the same image.
2. The **Black Top Hat (BTH)** is defined as the difference between a binary image and the opening of the same image.

WTH and BTH are also called morphological filters. They are adapted to extract linear and thin objects (darken or clear), of a given width. The following figures illustrate the two forms of the morphological filters.

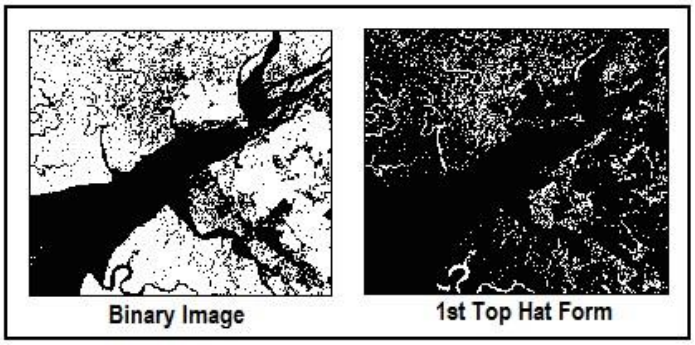


Figure 5.7 White Top Hat form of a SAR ERS-1 portion image of size 183x159 acquired on the region of Douala in Cameroon

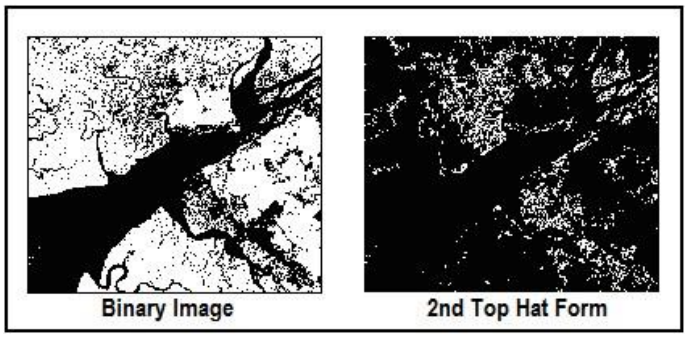


Figure 5.8 Black Top Hat form of a SAR ERS-1 portion image of size 183x159 acquired on the region of Douala in Cameroon

5.8 Image skeletisation

Binary image skeletisation consists in reducing the thickness of entities present in the image to the size of a pixel. The thinning process is conducted progressively, using a structuring element, until the stability is obtained. This process can be done by

convoluting successively the image with the following 8 masks. Each mask B_{j+1} is obtained by the rotation of B_j of 45°.

$$B1=\begin{bmatrix}0&0&0\\0&1&0\\1&1&1\end{bmatrix} \quad B2=\begin{bmatrix}0&0&0\\1&1&0\\1&1&0\end{bmatrix} \quad B3=\begin{bmatrix}1&0&0\\1&1&0\\1&0&0\end{bmatrix} \quad B4=\begin{bmatrix}1&1&0\\1&1&0\\0&0&0\end{bmatrix}$$

$$B5=\begin{bmatrix}1&1&1\\0&1&0\\0&0&0\end{bmatrix} \quad B6=\begin{bmatrix}0&1&1\\0&1&1\\0&0&0\end{bmatrix} \quad B7=\begin{bmatrix}0&0&1\\0&1&1\\0&0&1\end{bmatrix} \quad B8=\begin{bmatrix}0&0&0\\0&1&1\\0&1&1\end{bmatrix}$$

The skeletisation algorithm is the following.

Algorithm 5.2 Image_Skeletisation

Input: **IMG:** Binary image to skeletise

NL, NC: respectively the number of lines and the number of columns of IMG;

Output: **IMA:** Skeleton image;

Local variables: **i, j:** integers

IMA_{b1}, IMA_{b2}: Array[0..NL-1; 0..NC-1] of Booleans;

```
BEGIN
  FOR j varying from 0 to NL-1, DO
    FOR i varying from 0 to NC-1, DO
      BEGIN
        IMAb2[i,j]←FALSE;
        IF (IMG[i,j]>0) THEN IMAb1[i,j]←TRUE;
        ELSE IMAb1[i,j]←FALSE;
      END;
  Fill the borders of IMAb1 and IMAb2, using any of the three principles;
  Iter ← 1;
  Stable← False;
  WHILE (Stable=False) DO
      BEGIN
        Stable←True;
        Iter←Iter+1;
        //Convolution with the East Mask
        FOR j varying from 0 to NL-1, DO
          FOR i varying from 0 to NC-1, DO
```

$IF \quad (IMA_{b1}[j-1,i-1])OR(IMA_{b1}[j,i-1])OR(IMA_{b1}[j+1,i-1])OR \quad NOT(IMA_{b1}[j,i]$
$AND(IMA_{b1}[j-1,i+1])AND(IMA_{b1}[j,i+1])AND(IMA_{b1}[j+1,i+1])) \quad THEN$
$IMA_{b2}[j,i] \leftarrow IMA_{b1}[j,i]$
$ELSE$
$BEGIN$
$IMA_{b2}[j,i] \leftarrow False;$
$Stable \leftarrow False;$
END

//Convolution with the South-East Mask

FOR j varying from 0 to NL-1, **DO**
FOR i varying from 0 to NC-1, **DO**

$IF \quad (IMA_{b2}[j,i-1])OR(IMA_{b2}[j-1,i-1])OR(IMA_{b2}[j-1,i])OR \quad NOT(IMA_{b2}[j,i]$
$AND(IMA_{b2}[j,i+1])AND(IMA_{b2}[j+1,i])AND(IMA_{b2}[j+1,i+1])) \quad THEN$
$IMA_{b1}[j,i] \leftarrow IMA_{b2}[j,i]$
$ELSE$
$BEGIN$
$IMA_{b1}[j,i] \leftarrow False;$
$Stable \leftarrow False;$
END

//Convolution with the South Mask

FOR j varying from 0 to NL-1, **DO**
FOR i varying from 0 to NC-1, **DO**

$IF \quad (IMA_{b1}[j-1,i-1])OR(IMA_{b1}[j-1,i])OR(IMA_{b1}[j-1,i+1])OR \quad NOT(IMA_{b1}[j,i]$
$AND(IMA_{b1}[j+1,i-1])AND(IMA_{b1}[j+1,i])AND(IMA_{b1}[j+1,i+1])) \quad THEN$
$IMA_{b2}[j,i] \leftarrow IMA_{b1}[j,i]$
$ELSE$
$BEGIN$
$IMA_{b2}[j,i] \leftarrow False;$
$Stable \leftarrow False;$
END

//Convolution with the South-West Mask

FOR j varying from 0 to NL-1, **DO**
FOR i varying from 0 to NC-1, **DO**

$IF \quad (IMA_{b2}[j-1,i+1]) OR (IMA_{b2}[j,i+1]) OR (IMA_{b2}[j,i+1]) OR \quad NOT(IMA_{b2}[j,i]$
$AND(IMA_{b2}[j,i-1]) AND(IMA_{b2}[j+1,i-1]) AND(IMA_{b2}[j+1,i])) \quad THEN$
$IMA_{b1}[j,i] \leftarrow IMA_{b2}[j,i]$
$ELSE$
$BEGIN$
$IMA_{b1}[j,i] \leftarrow False;$
$Stable \leftarrow False;$
END

//Convolution with the West Mask

FOR j varying from 0 to NL-1, **DO**
FOR i varying from 0 to NC-1, **DO**

$IF \quad (IMA_{b1}[j-1,i+1]) OR (IMA_{b1}[j,i+1]) OR (IMA_{b1}[j+1,i+1]) OR \quad NOT(IMA_{b1}[j,i]$
$AND(IMA_{b1}[j-1,i-1]) AND(IMA_{b1}[j,i-1]) AND(IMA_{b1}[j+1,i-1])) \quad THEN$
$IMA_{b2}[j,i] \leftarrow IMA_{b1}[j,i]$
$ELSE$
$BEGIN$
$IMA_{b2}[j,i] \leftarrow False;$
$Stable \leftarrow False;$
END

//Convolution with the North-West Mask

FOR j varying from 0 to NL-1, **DO**
FOR i varying from 0 to NC-1, **DO**

$IF \quad (IMA_{b2}[j,i+1]) OR (IMA_{b2}[j+1,i+1]) OR (IMA_{b2}[j+1,i]) OR \quad NOT(IMA_{b2}[j,i]$
$AND(IMA_{b2}[j-1,i]) AND(IMA_{b2}[j-1,i-1]) AND(IMA_{b2}[j,i-1])) \quad THEN$
$IMA_{b1}[j,i] \leftarrow IMA_{b2}[j,i]$
$ELSE$
$BEGIN$
$IMA_{b1}[j,i] \leftarrow False;$
$Stable \leftarrow False;$
END

//Convolution with the North Mask

FOR j varying from 0 to NL-1, **DO**
FOR i varying from 0 to NC-1, **DO**

$IF \quad (IMA_{b1}[j+1,i-1])OR(IMA_{b1}[j+1,i])OR(IMA_{b1}[j+1,i+1])OR \quad NOT(IMA_{b1}[j,i]$
$AND(IMA_{b1}[j-1,i-1])AND(IMA_{b1}[j-1,i])AND(IMA_{b1}[j-1,i+1])) \quad THEN$
$IMA_{b2}[j,i] \leftarrow IMA_{b1}[j,i]$
ELSE
BEGIN
$IMA_{b2}[j,i] \leftarrow False;$
$Stable \leftarrow False;$
END

//Convolution with the North-East Mask

FOR j varying from 0 to NL-1, **DO**
FOR i varying from 0 to NC-1, **DO**

$IF \quad (IMA_{b2}[j,i-1])OR(IMA_{b2}[j+1,i-1])OR(IMA_{b2}[j+1,i])OR \quad NOT(IMA_{b2}[j,i]$
$AND(IMA_{b2}[j-1,i])AND(IMA_{b2}[j-1,i+1])AND(IMA_{b2}[j,i+1])) \quad THEN$
$IMA_{b1}[j,i] \leftarrow IMA_{b2}[j,i]$
ELSE
BEGIN
$IMA_{b1}[j,i] \leftarrow False;$
$Stable \leftarrow False;$
END

END

FOR j varying from 0 to NL-1, **DO**
FOR i varying from 0 to NC-1, **DO**

IF (IMAb1[j,i]) **THEN** IMA[j,i]←255;

ELSE IMA[j,i]←0;

END

The following figure gives an example of skeletisation applied on the binary image of figure 5.2.

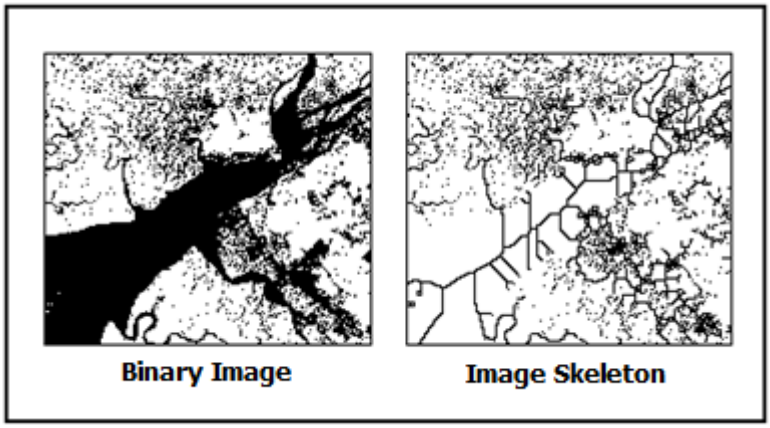


Figure 5.9 Skeletisation of a portion of a SAR image acquired by ERS-1 on the Cameroonian Littoral region.

CHAPTER 6. TEXTURE ANALYSIS

6.1 Introduction

Distinct objects can have similar or identical spectral properties. This situation generally generates confusions between thematic classes if the classification process is based only on the spectral information contained in the image. This is why classification methods based on spatial information have been developed, notably through texture analysis. Texture analysis is a method that takes into account the spatial distribution of greyscales in a given pixel neighbourhood in the image. Statistical methods are defined with respect to various orders.

6.2 First order statistical methods

6.2.1. Definition

First order statistical methods are based on first order histograms. Such a histogram specifies the frequency of appearance of a given greyscale in a given neighbourhood of the image called **image window**. An image window is a rectangular or square portion of the image. From the histogram, one can extract various statistical parameters. The following section illustrates some of them.

6.2.2 Examples of first order statistical parameters

Let consider an image window W of size W_x colums and W_y lines. The probability P(i) of occurrence of the greyscale i in W is given by $P(i) = N(i) / (W_x * W_y)$, where N(i) is the number of greyscales i in W. Let denote by *MaxGs* the maximum greyscale in W. The following equations define some statistical parameters of order 1.

Average or Mean: $$S_M = \sum_{i=0}^{MaxGs} i \cdot P(i) \qquad \text{(Eq. 6.1)}$$

This parameter represents the location of the histogram on the scale of pixel greyscales.

Entropy: $$Ent = -\sum_{i=0}^{MaxGs} P(i) \cdot Log_2[P(i)] \qquad \text{(Eq. 6.2)}$$

This parameter measures the image complexity. In other words, it measures the degree of disorder in the image.

Standard Deviation: $$S_D = \sqrt{\sum_{i=0}^{MaxGs} (i - S_M)^2} \quad \text{(Eq. 6.3)}$$

This parameter measures the organisation of greyscales around the Average.

Skewness: $$Skw = \frac{1}{S_D^{\,3}} \cdot \left[\sum_{i=0}^{MaxGs} (i - S_M)^3 \cdot P(i) \right] \quad \text{(Eq. 6.4)}$$

This parameter measures the histogram dissimilarity. i.e. if it is well equilibrated around the Average or if it is left or right oriented with respect to the mean.

Kurtosis: $$Kurt = \frac{1}{S_D^{\,4}} \cdot \left[\sum_{i=0}^{MaxGs} (i - S_M)^4 \cdot P(i) \right] \quad \text{(Eq. 6.5)}$$

This parameter gives an indication on the histogram aspect. i.e. if it is flatten or not.

6.2.3. Illustration

Let calculate for example the parameter Mean for the following simple digital image IMG specified on the left of the following figure. Let calculate the values of P(i) for all the greyscales of the image, using a 3x3 size image window and the symmetry principle of border filling.

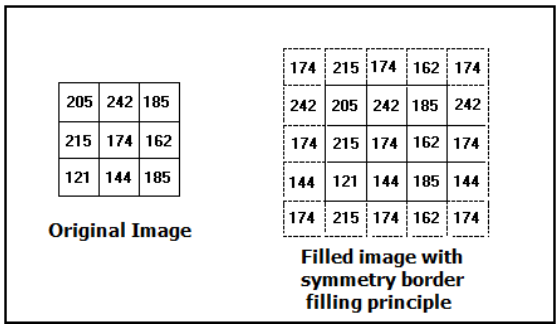


Figure 6.1. Original image and border filled image with the principle of symmetry.

From the first pixel IMG_{00} (greyscale 205), the following image window is extracted.

$$W_{205} = \begin{bmatrix} 174 & 215 & 174 \\ 242 & 205 & 242 \\ 174 & 215 & 174 \end{bmatrix}$$

We have: $W_x = W_y = 3$;

$P(174) = N(174)/(3*3) = 4/9$; $P(215) = N(215)/M = 2/9$;

$P(242) = N(242)/(3*3) = 2/9$; $P(205) = N(205)/M = 1/9$;

The value of the parameter **Mean** related to the greyscale 205 is given by:

$S_M(205)$ = **ENT**[(174x4/9)+(215x2/9)+(242x2/9)+(205x1/9)] = ENT[201.66] = 201

Where **ENT** is the entire part function.

The process is repeated with the same principle to other pixels of the original image IMG and, considering that W_{ij} is an image window of size 3*3 centred on the pixel (i,j) of IMG, we obtain the following results:

$S_M(W_{01}) = 192$; $S_M(W_{02}) = 187$; $S_M(W_{10}) = 181$; $S_M(W_{11}) = 183$;

$S_M(W_{12}) = 172$; $S_M(W_{20})$; $= 165$ $S_M(W_{21}) = 184$; $S_M(W_{22}) = 170$.

The following figure presents the image texture of the parameter Mean obtained.

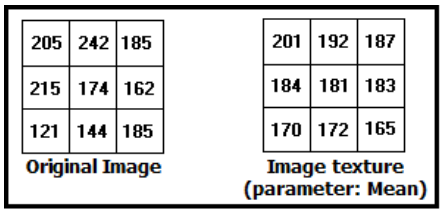


Figure 6.2. Example of image texture: The parameter Mean

6.3 Second order statistical methods

6.3.1 Definition

These methods are based on the second order histograms called co-occurrence matrices. The co-occurrence matrix expresses the probability of appearance of each couple of greyscales (i,j) in the image window, with respect to a given direction and inter-pixels distance. Below are some commonly used order-2 texture parameters.

***Energy or second angular moment*:** $$E = \sum_{i=0}^{MaxGs} \sum_{j=0}^{MaxGs} [P(i,j,d,\theta)]^2 \qquad \text{(Eq. 6.6)}$$

The energy parameter measures the homogeneity of the image. Its value is low when the values of the co-occurrence matrix are nearer. In contrary, if some values of the co-occurrence matrix are very bigger than others, the value of the energy becomes high.

Entropy: $$Ent = \sum_{i=0}^{MaxGs} \sum_{j=0}^{MaxGs} P(i,j,d,\theta) \cdot \log_2(P(i,j,d,\theta)) \qquad \text{(Eq. 6.7)}$$

The entropy is great when the values of the co-occurrence matrix are almost equal each to another. Otherwise, its value is low.

Contrast or inertia: $$Cont = \sum_{i=0}^{MaxGs} \sum_{j=0}^{MaxGs} (i-j)^2 P(i,j,d,\theta) \qquad \text{(Eq. 6.8)}$$

The inertia parameter measures the contrast (or the local variation of greyscales) in the image. Its value is great when the values of the co-occurrence matrix are concentrated far from the diagonal. If in contrary the values of the co-occurrence matrix are concentrated around the diagonal then the contrast is weak.

Correlation: $$Corr = \sum_{i=0}^{MaxGs} \sum_{j=0}^{MaxGs} \left[(i-\mu_x)(j-\mu_y)P(i,j,d,\theta)\right] / \left(\sigma_x \sigma_y\right) \quad \text{(Eq. 6.9)}$$

The correlation has a great value when the values of the co-occurrence matrix are uniformly distributed. Otherwise, its value is low.

Cluster prominence: $$Cpr = \sum_{i=0}^{MaxGs} \sum_{j=0}^{MaxGs} \left[\left(i + j - \mu_x - \mu_y\right)^4 P(i,j,d,\theta) \right] \quad \text{(Eq. 6.10)}$$

Where:

- $P(i,j,d,\theta)$ is the probability of appearance of the couple of greyscales (i,j) in the direction θ, with an inter-pixels distance d.
- μ_x and σ_x represent respectively the mean and the standard deviation with respect to the lines of the co-occurrence matrix.
- μ_y and σ_y represent respectively the mean and the standard deviation with respect to the columns of the co-occurrence matrix.

6.3.2 Co-occurrence matrix

The co-occurrence matrix is defined with respect to a given direction. In what follows, algorithms of the co-occurrence matrix for the four principal directions, notably 0°, 45°, 90° and 135° are presented.

Algorithm 6.1. CooccurrenceMatrix_0°_Direction

Input:

W: Image window of size W_x columns and W_y lines;
MaxGs: Maximum greyscale in the image;
d: inter-pixel distance;

Output:

CM: Co-occurrence matrix of size (MaxGs+1)*(MaxGs+1);

Local Variables:

i, j, x, y: Integers;

```
Begin
  For j varying from 0 to MaxGs do
    For i varying de 0 to MaxGs do
        CM[j,i] ← 0 ;  //Initialisation
  For y varying from 0 to Wy-1 do
   For x varying from 0 to Wx-1-d do
     Begin
       I ← W[y,x]
       J ← W[y,x+d]
       CM [I,J] ← CM [I,J] + 1;
       CM [I,J] ← CM [J,I] + 1;
     End
  For j varying from 0 to MaxGs do
    For i varying from 0 to MaxGs do
        MC [i,j] ← MC [j,i]/(Wy*(Wx-d));
End
```

Algorithm 6.2. CooccurrenceMatrix_45°_Direction

Input:

W: Image window of size W_x columns and W_y lines;
MaxGs: Maximum greyscale in the image;
d: inter-pixel distance;

Output:

CM: Co-occurrence matrix of size (MaxGs+1)*(MaxGs+1);

Local Variables:

i, j, x, y: Integers;

```
Begin
  For j varying from 0 to MaxGs do
    For i varying de 0 to MaxGs do
        CM[j,i] ← 0 ;  //Initialisation
  For y varying from 0 to Wy-1-d do
   For x varying from 0 to Wx-1-d do
     Begin
       I ← W[y,x]
       J ← W[y+d,x+d]
       CM [I,J] ← CM [I,J] + 1;
       CM [I,J] ← CM [J,I] + 1;
     End
  For j varying from 0 to MaxGs do
    For i varying from 0 to MaxGs do
        MC [i,j] ← MC [j,i]/((Wy-d)*(Wy-d));
End
```

Algorithm 6.3. CooccurrenceMatrix_90°_Direction

Input:

W: Image window of size W_x columns and W_y lines;
MaxGs: Maximum greyscale in the image;
d: inter-pixel distance;

Output:

CM: Co-occurrence matrix of size (MaxGs+1)*(MaxGs+1);

Local Variables:

i, j, x, y: Integers;

```
Begin
  For j varying from 0 to MaxGs do
    For i varying de 0 to MaxGs do
        CM[j,i] ← 0 ;  //Initialisation
  For y varying from d to Wy-1 do
   For x varying from 0 to Wx-1 do
     Begin
       I ← W[y,x]
       J ← W[y-d,x]
       CM [I,J] ← CM [I,J] + 1;
       CM [I,J] ← CM [J,I] + 1;
     End
  For j varying from 0 to MaxGs do
    For i varying de 0 to MaxGs do
        MC [i,j] ← MC [j,i]/(Wx*(Wy-d));
End
```

Algorithm 6.4. CooccurrenceMatrix_135°_Direction

Input:

W: Image window of size W_x columns and W_y lines;
MaxGs: Maximum greyscale in the image;
d: inter-pixel distance;

Output:

CM: Co-occurrence matrix of size (MaxGs+1)*(MaxGs+1);

Local Variables:

i, j, x, y: Integers;

```
Begin
  For j varying from 0 to MaxGs do
    For i varying de 0 to MaxGs do
        CM[j,i] ← 0 ;  //Initialisation
  For y varying from d to Wy-1 do
   For x varying from d to Wx-1 do
     Begin
       I ← W[y,x]
       J ← W[y-d,x-d]
       CM [I,J] ← CM [I,J] + 1;
       CM [I,J] ← CM [J,I] + 1;
     End
  For j varying from 0 to MaxGs do
    For i varying from 0 to MaxGs do
        MC [i,j] ← MC [j,i]/((Wy-d)*(Wx-d));
End
```

6.3.3 Image texture

In this section, we apply directly the principle on an example. Let consider for example the parameter **Energy**. The following algorithm specifies the technique of **Energy image texture** construction.

Algorithm 6.5 Energy_Image_Texture

Input: IMG: Original image of size NL lines and NC columns;
MaxGs: maximum greyscale in IMG;
Θ, *d*: respectively the direction and the inter-pixel distance;

Output: **ENR_IMG:** Energy texture image with the same size as IMG;

Local Variables: x,y,i,j: **integers**;
IMA: Array[0..NL-1; 0..NC-1] of reals;
E, E_Max: reals;

```
BEGIN
  FOR y varying from 0 to NL-1 DO
    FOR x varying from 0 to NC-1 DO
      Begin
        1- Extract an image window of size Wx columns and Wy lines centred on the
           pixel (y,x) of IMG;
        2- Calculate the co-occurrence matrix CM related to W, in the direction Θ and
           the inter-pixel distance d;
        3- E ← 0; E_Max ← 0;
        4- FOR j varying from 0 to MaxGs DO
             FOR i varying from 0 to MaxGs DO
               E ← E+(CM[j,i]*CM[j,i]);
        5- IMA[y,x] ← E;
        6- IF (E > E_max) THEN E_max ← E;
      End
  FOR y varying from 0 to NL-1 DO
    FOR x varying from 0 to NC-1 DO
      ENR_IMG[y,x] ← ENT((255* IMA[y,x])/E_max);
END
```

ENT is the entire part function.

It is also possible to consider all the directions at once, either by taking the average value of the co-occurrence matrices in various directions, or by taking the maximum value of the co-occurrence matrix.

6.3 Grayscale ranges lengths method

The method of grayscale ranges lengths considers an auto-adaptive and mono-dimensional neighbourhood. A point belongs to a neighbourhood of the current point if its greyscale is less different. The size of the neighbourhood is then variable and constitutes a major characteristic parameter in this approach. The grayscale ranges length method consists in counting, in the image, the number of ranges of greyscales of a given length, in a given direction. A matrix R(θ) of grayscale ranges lengths is associated to a direction of angle θ. From this matrix, the following parameters can be extracted.

- Short Run Low Greyscale Emphasis $SRLGE = \frac{1}{nr}\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{r(i,j;\theta)}{i^2 j^2}$ (Eq. 6.11)
- Short Run High Greyscale Emphasis $SRHGE = \frac{1}{nr}\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{i^2 r(i,j;\theta)}{j^2}$ (Eq. 6.12)
- Long Run Low Greyscale Emphasis $LRLGE = \frac{1}{nr}\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{j^2 r(i,j;\theta)}{i^2}$ (Eq. 6.13)
- Long Run High Greyscale Emphasis $LRHGE = \frac{1}{nr}\sum_{i=1}^{M}\sum_{j=1}^{N} i^2 j^2 r(i,j;\theta)$ (Eq. 6.14)
- Distribution of greyscales $DGS = \frac{1}{nr}\sum_{i=1}^{M}\left[\sum_{j=1}^{N} r(i,j;\theta)\right]^2$ (Eq. 6.15)
- Distribution of lengths of ranges $DLR = \frac{1}{nr}\sum_{j=1}^{N}\left[\sum_{i=1}^{M} r(i,j;\theta)\right]^2$ (Eq. 6.16)
- Greyscales Percentage $GSP = \frac{1}{NB}\sum_{i=1}^{M}\sum_{j=1}^{N} r(i,j;\theta)$ (Eq. 6.17)
- Low greyscale range $LSGSR = \frac{1}{nr}\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{r(i,j;\theta)}{i^2}$ (Eq. 6.18)
- High greyscale range $HGR = \frac{1}{nr}\sum_{i=1}^{M}\sum_{j=1}^{N} i^2 \times r(i,j;\theta)$ (Eq. 6.19)

The constant $nr = \sum_{i=1}^{M}\sum_{j=1}^{N} r(i,j;\theta)$ is the total number of ranges in the image window, with respect to the orientation Θ, centred on the current pixel. M and N are respectively the number of greyscales and ranges in the image window. r[] is the matrix of wavelength ranges in a given direction. Its specification with respect to the four principal directions is the following.

Algorithm 6.6 Grayscale_Ranges_Lengths_Matrix_0°
Input:
W: Image window of size W_x columns and W_y lines;
Range: Array[0..M_{ax}GS] of bytes; // array of grayscale ranges in the image window W.
Output:
WR: Array [0..M_{ax}GS;1..M_{ax}R]of integers;// Matrix of greyscale ranges lengths in 0° direction;
Local variables:
i, j, a, b, N: integers;
X,R: Bytes;
BEGIN
Initialise WR to 0;
FOR j varying from 0 to W_y-1 **DO**
Begin
X ← W[j,0];
R ← Range[X];
N ← 1;
i ← 1;
WHILE (i<W_x) **DO**
Begin
X ← W[j,i];
i ← i+1;
IF (Range[X]=R) **THEN**
N ← N+1;
ELSE
WR[R,N] ← WR[R,N]+1;
R ← Range[X];
N ← 0;
END_IF
END_WHILE
END_FOR
END

Algorithm 6.7 Grayscale_Ranges_Lengths_Matrix_45°
Input:
W: Image window of size W_x columns and W_y lines;
Range: Array[0..M_{ax}GS] of bytes; // array of grayscale ranges in the image window W.
Output:
WR: Array [0..M_{ax}GS;1..M_{ax}R]of integers;// Matrix of greyscale ranges lengths in 45° direction;
Local variables:
i, j, a, b, N: integers;
X,R: Bytes;
BEGIN
Initialise WR to 0;
FOR j varying from 0 to W_y-1 **DO**
Begin
X ← W[j,0];
R ← Range[X];
N ← 1;
b ← j;
FOR i varying from 1 to Minimum(j,W_x-1) **DO**
Begin

```
            X ← W[b-1,i];
            b ← b-1;
            IF (Range[X]=R) THEN
              N ← N+1;
            ELSE
              Begin
                WR[R,N] ← WR[R,N]+1;
                R ← Range[X];
                N ← 0;
              END_IF
          END_FOR
      END_FOR
  FOR i varying from 1 to Wx-1 DO
      Begin
        X ← W[Wy-1,i];
        R ← Range[X];
        N ← 1;
        b ← Wy-1;
        FOR a varying from 0 to Minimum(Wy-1,i) DO
          Begin
            X ← W[b-1,a];
            b ← b-1;
            IF (Range[X]=R) THEN
              N ← N+1;
            ELSE
              Begin
                WR[R,N] ← WR[R,N]+1;
                R ← Range[X];
                N ← 0;
              END_IF
          END_FOR
      END_FOR
END
```

Algorithm 6.8 Grayscale_Ranges_Lengths_Matrix_90°
Input:
W: Image window of size W_x columns and W_y lines;
Range: Array[0..M_{ax}GS] of bytes; // array of grayscale ranges in the image window W.
Output:
WR: Array [0..M_{ax}GS;1..M_{ax}R]of integers;// Matrix of greyscale ranges lengths in 90° direction;
Local variables:
i, j, a, b, N: integers;
X,R: Bytes;
BEGIN
Initialise WR to 0;
FOR i varying from 0 to W_x-1 **DO**
Begin
X ← W[0,i];
R ← Range[X];
N ← 1;
j ← 1;
WHILE (j<W_y) **DO**
Begin
X ← W[j,i];
j ← j+1;
IF (Range[X]=R) **THEN**
N ← N+1;
ELSE
WR[R,N] ← WR[R,N]+1;
R ← Range[X];
N ← 0;
END_IF
END_WHILE
END_FOR
END

Algorithm 6.9 Grayscale_Ranges_Lengths_Matrix_135°
Input:
W: Image window of size W_x columns and W_y lines;
Range: Array[0..M_{ax}GS] of bytes; // array of grayscale ranges in the image window W.
Output:
WR: Array [0..M_{ax}GS;1..M_{ax}R]of integers;// Matrix of greyscale ranges lengths in 135° direction;
Local variables:
i, j,a,b, N: integers;
X,R: Bytes;
BEGIN
Initialise WR to 0;
FOR j varying from 0 to W_y-1 **DO**
Begin
X ← W[j,W_x-1];
R ← Range[X];
N ← 1;
b ← j;
FOR i varying from 1 to Minimum(j,W_x-1) **DO**
Begin

```
            X ← W[b-1, Wx-1-i];
            b ← b-1;
            IF (Range[X]=R) THEN
              N ← N+1;
            ELSE
              Begin
                WR[R,N] ← WR[R,N]+1;
                R ← Range[X];
                N ← 0;
              END_IF
          END_FOR
      END_FOR
  FOR i varying from Wx-2 down to 0 DO
      Begin
        X ← W[Wy-1,i];
        R ← Range[X];
        N ← 1;
        b ← Wy-1;
        FOR a varying from 1 to Minimum(Wy-1,i) DO
          Begin
            X ← W[b-1, i-a];
            b ← b-1;
            IF (Range[X]=R) THEN
              N ← N+1;
            ELSE
              Begin
                WR[R,N] ← WR[R,N]+1;
                R ← Range[X];
                N ← 0;
              END_IF
          END_FOR
      END_FOR
END
```

Where:

- MaxGS is the maximum greyscale in the image window W;
- MaxR is the maximum length of a range in the image window. It is defined as follows:
  - MaxR=NL if direction=90°;
  - MaxR=NC if direction=0°;
  - MaxR=Minimum(NL;NC) otherwise.

Sometime, it is important to resample the image for the execution time optimisation purpose. Image resampling consists in reducing uniformly the values of the greyscales present in the image. Following is the related algorithm.

Algorithm 6.9 Image_Resample

Input:

IMG: Original image of NL lines and NC columns;

M_{ax}GS: Byte representing the new value of the maximum greyscale in the resampled image;

Output:

IMA: Resampled image of NL lines and NC columns;

Local variable:

i,j: integers;

M_{ax}Old: byte representing the maximum greyscale in the original image;

BEGIN

M_{ax}Old ← 0;

FOR j varying from 0 to NL **DO**

FOR i varying from 0 to NC **DO**

IF IMG[j,i]>MaxOld **THEN** MaxOld ← IMG[j,i];

FOR j varying from 0 to NL **DO**

FOR i varying from 0 to NC **DO**

IMA[j,i] ← **ENT**(IMG[j,i]*(M_{ax}GS/MaxOld));

END

Once the matrix of greyscale ranges lengths is calculated, it is now easy to calculate the image texture for a parameter such as the Short Run Low Grayscale Emphasis (SRLGE).

Algorithm 6.10 ImageTexture_SRLGE

Input:

IMG: Original image of size NC columns and NL lines;

W_x, W_y: integers representing respectively the number of columns and lines of the image window;

θ: inter-pixels direction;

Output:

IMA: Image texture for the parameter SRLGE;

Local variable:

Ima_f: Array[0..NL-1;0..NC-1] of reals;

a, $SRLGE\_{Max}$: real;

i, j: integers;

W: Image window of size W_x columns and W_y lines;

MR_θ: Matrix of grayscale ranges lengths;

```
    NR: integer representing number of ranges of grayscales in the image window W;
BEGIN
    SRLGE_Max ← 0;
    FOR y varying from 0 to NL-1 DO
      FOR x varying from 0 to NC-1 DO
        Begin
          Extract and image window W centred on the pixel (y,x) of the image IMG;
          Calculate the matrix MRθ of wavelength ranges of greyscales related to W in the direction θ;
          a ← 0;
          FOR i varying from 0 to MaxGS, DO
            FOR j varying from 1 to NR DO
              a ← a+(MRθ(i, j)/[(i+1)² × (j+1)²]);
          a ← a/NR;
          Imaf[y,x] ← a;
          IF(a > SRLGE_Max) THEN SRLGE_Max ← a;
        End
    FOR y varying from 0 to NL-1 DO
      FOR x varying from 0 to NC-1 DO
        Begin
          Imaf[y,x] ← ENT((255* imaf[x,y])/SRLGE_Max);
END
```

Where ENT is the entire part function.

CHAPTER 7. IMAGE CLASSIFICATION

7.1 Introduction

Remote sensing data can be analysed for the extraction of useful thematic information. The multispectral classification is one of the most used for information retrieval. This technique supposes that images of a given geographical region are distributed in different electromagnetic spectrum. A multispectral classification can be done by the means of various algorithms among which:

- Supervised statistical or neuronal classification methods;
- Unsupervised statistical or neuronal classification methods;
- Fuzzy classification methods
- Expert systems or knowledge-based classification methods.
- Hybrid methods based on the previous methods.

The supervised classification method supposes that the analyst has some knowledge of the field from which the image was acquired. This knowledge obtained through field mission or validated maps of the area of study helps to identify training sites on the image. These training sites are used to train the classifier before the automation of the process. Each training site characterises an information or thematic class such as, Lake, Forest, Road, Crops, etc.

In contrary to the supervised methods, the unsupervised (non-supervised) method supposes that the analyst doesn't have any knowledge of the area. i.e no field mission has been conducted on the area of study and there is no available map of the region of study. The analyst just specifies the number of spectral classes and the classifier, based on spectral information, tries its best to classify the image. These spectral classes are interpreted by the analyst to produce information classes.

The fuzzy classification method considers the heterogeneous and inaccurate nature of geographical data. It can be either based on supervised or non-supervised algorithms. Most of classification algorithms use Boolean theory of sets to produce homogeneous information classes, completely ignoring the inaccuracy nature of geographical data. Instead of assigning each pixel of the image to a unique class among M others, the fuzzy classification calculates M real numbers indicating the degree of nearness of the pixel to

each class of interest. This information is afterwards used by the analyst to extract more accurate information on the area of study such as the heterogeneous pixels composition. Sometimes, it is necessary to include auxiliary and non-spectral data in a fuzzy supervised or unsupervised classification to extract expected information. Among such methods, one can distinguish hierarchical, expert systems and knowledge-based classifications.

7.2 Classification process

7.2.1 Classification protocol

A good classification (supervised or non-supervised) of remote sensing data is tributary to the mastering of the following steps:

1) Identification and selection of a region of interest (ROI) on which hypothesis should be tested. The classes of interest to be tested in the hypothesis determine the nature of the classification method to use.

2) Selection of the appropriate remote sensing images, considering environmental and data acquisition system constraints. Based on these constraints, radiometric and/or geometric corrections are done on images finally acquired.

3) Choice of the appropriate classification algorithm and collection of initial training data.

4) Identification of statistical parameters that better discriminate the training data.

5) Training of the classifier with the parameters selected in the previous step, in case of a supervised classification.

6) Collection of additive training data for the evaluation of the classifier performances. If the performances are acceptable, the classifier is adopted for the global image classification.

7) Evaluation of the classification error. If the result is satisfactory then the classification maps are validated.

7.2.2 Classification system

There is a great difference between an information class present on the field and a spectral class contained in an image. Information classes are categories of interest for data users, such as geological, forest and crops units that give information to the manager, administrators and scientists in charge of remote sensing data.

Spectral classes are groups of pixels that have similar spectral characteristics directly observable on the image. An image becomes a good source of information when there is a link between spectral classes observable on the image and information classes present on the field. Thus, remote sensing classification consists in establishing a correspondence between spectral and information classes. When this correspondence is accurate, the resulting classification maps are reliable. Otherwise, the image is not a good source of information.

In the same information class, spectral characteristics can vary, due to the natural environment. For example, in a Forest information class, spectral characteristics can change with the type and age of trees. So the same information class can contain many spectral classes. Similarly, two or more different information classes can have the same spectral information. For example, under a certain wavelength, the spectral information of the Lake is similar to the spectral information of a targeted road.

7.2.3 Training sites selection and statistics extraction

Training sites in remote sensing must be representative of land cover, after the classification system is adopted. The quality of training data depends on the homogeneity of the environment. If the land characteristics varies along the region of study then the spectral properties of the training data collected should not be homogeneous, generating the problem of spectral signatures extension. In this case, following are some factors that help to solve the problem:

- Differences on soil types;
- Water turbidity;
- Types of crops;
- Unusual soil humidity conditions, eventually caused by non-uniform precipitations;
- diffuser plates of atmospheric gases;

Once extension factors of spectral signatures are considered, training sites are selected and characterised for each information class. In case of n spectral bands (multiband image), each pixel of the training site is represented by an array X_c of measures defined by the following equation.

$$X_c^m = \begin{pmatrix} GV_{ij1}^m \\ \cdot \\ \cdot \\ \cdot \\ GV_{ij1}^m \end{pmatrix} \quad \text{(Eq. 6.1)}$$

Where GV_{ij1}^m is the grayscale of the m[th] pixel at location (i,j) in band k. Combining all the N pixels of the training site of an information class, the characteristic vector M_c of the information class is the following.

$$M_c = \begin{pmatrix} \mu_{c1} \\ \cdot \\ \cdot \\ \cdot \\ \mu_{cn} \end{pmatrix} \quad \text{(Eq. 6.2)}$$

Where μ_{ck} is the average value of the characteristic vectors of the N training pixels selected in the class C_k. These characteristic vectors can also be analysed to provide the following characteristic covariance matrix:

$$V_c = \begin{pmatrix} Cov_{cll} & \cdot & \cdot & \cdot & Cov_{cnl} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ Cov_{cnl} & \cdot & \cdot & \cdot & Cov_{cnn} \end{pmatrix} \quad \text{(Eq. 6.3)}$$

Where Cov_{ckl} is the covariance of class C between bands k and l.

7.2.4 Reduction of parameters

The representation of input data of a static or neuronal classification system is very important. It can significantly affect the performances of the system. There are two approaches of parameters reduction: parameters extraction and parameters selection.

Parameters extraction is an operation that consists in finding a set of vectors representing the observation by reducing the dimensionality. Despite the fact that the reduction of the dimensionality is useful, the inducted error must not scarify the discrimination power of the classifier. Methods of parameters extraction can be supervised or non-supervised, linear or non-linear. Among these methods, one can distinguish the principal components analysis. For all these methods, a characteristic matrix is defined. From this matrix, eigenvalues and corresponding eigenvectors are calculated and decreasingly ordered. The dimension of the input data corresponds to the number of eigenvectors selected.

Transformed data are determined by $Y = \phi^T X$, where ϕ is the transformation matrix made of eigenvectors of the characteristic matrices, X is the input data vector in the original characteristic space and Y is the output data vector in the new characteristic space.

7.3 Supervised image classification algorithms

A supervised image classification algorithm consists in three steps:

1) **Training step:** For each information class identified in the image, a certain number of pixels are selected and characterised with the perspective of training the classifier.
2) **Information Classes allocation step:** Each pixel of the pixel is assigned to information class with which the likelihood is maximum.
3) **Verification step:** The quality of classification s evaluated.

Following are some commonly used supervised classification algorithms:

- Parallelepiped decision rule;
- Minimal distance decision rule;
- k nearest neighbours decision rule;
- Maximum likelihood decision rule.

To these methods, one must add classification methods based on artificial neuronal networks.

7.3.1 Parallelepiped method for image classification

This Boolean logic method-based method is widely used. Training data in n spectral bands are used in the classification process. For each information class C, the characteristic vector V_c is calculated. $V_c = \begin{pmatrix} \mu_{c1} \\ \cdot \\ \cdot \\ \cdot \\ \mu_{cn} \end{pmatrix}$ Where μ_{ck} represents the average greyscale of selected pixels in the information class C in bad k. Let σ_{ck} be the standard deviation of training data of the information class C in band k. Considering σ_{ck} as the classification threshold

for example, the parallelepiped algorithm assigns a pixel characterised by the vector $X_c = \begin{pmatrix} GV_{ij1} \\ \cdot \\ \cdot \\ \cdot \\ GV_{ijn} \end{pmatrix}$ to the information class C if and only if the following equation is satisfied.

$$\mu_{ck} - \sigma_{ck} \le GV_{ijk} \le \mu_{ck} + \sigma_{ck}, \quad \forall k \qquad \text{(Eq. 6.4)}$$

If the lower and upper decision boundaries are defined respectively by $L_{ck} = \mu_{ck} - \sigma_{ck}$ and $U_{ck} = \mu_{ck} + \sigma_{ck}$ then the parallelepiped decision rule is expressed as follows:

$$L_{ck} \le GV_{ijk} \le U_{ck}, \quad \forall k \quad \Rightarrow \quad BV_{ij} \in w_c \qquad \text{(Eq. 6.5)}$$

This decision rule constitute a multidimensional parallelepiped in the characteristic space. If the value of a pixel is between the lower and upper thresholds of an information class for all the bands then the pixel is assigned to the class. If this condition is not followed for any of the n information classes then the pixel is said to be non-classified.

7.3.2 Minimum distance method for image classification

The minimum distance method for image classification is similar to the parallelepiped method. It takes the average characteristic vectors V_c of all the classes obtained from the training data. For each pixel located at (i,j) in the image, the classifier calculates the characteristic vector BV in each band and calculates the following distance:

$$D_{cij} = \sqrt{\sum_{k=1}^{n} \left(GV_{ijk} - \mu_{ck}\right)^2} \qquad \text{(Eq. 6.6)}$$

The class C that produces minimises this distance is the class to which belongs the pixel (i,j) as specified in the following equation.

$$\mathrm{D_{cij}} = \min_{\mathrm{k}} \left\{ \mathrm{D_{kij}} \right\} \quad \Rightarrow \quad \mathrm{X_{ij}} \in \mathrm{w_c} \qquad \text{(Eq. 6.7)}$$

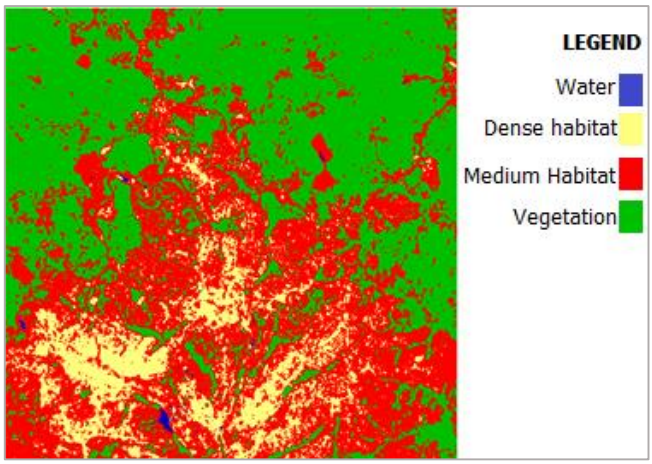


Figure 7.1. Example of a minimum distance image classification applied on a SPOT image acquired on the Yaounde town.

7.3.3 K nearest neighbours method for image classification

As the previous method, the K nearest neighbours method is based on the characteristic vectors $(X_a)_{1\le a\le n}$ of the training pixels, N being the number of training pixels in the information class X. The classifier calculates the Euclidian distance between the characteristic vector $(GV_{ijk})_{1\le k\le n}$ of each unknown pixel (i,j) and each characteristic vector X_i of the training pixel i, n being the number of bands. The process end by determining appearance frequency f_{ck} of pixels of each class C between the K nearest pixels of the considered pixel. The following equation formalises the process.

$$f_{ck} = \max_c \{f_{ck}\} \quad \Rightarrow \quad GV_{ij} \in w_k \qquad \text{(Eq. 6.8)}$$

It is obviously required that K should be defined. Practically, The value of K is given by $K = c\sqrt{N}$, where c is an appropriate constant (typically, c=1) and N is the total number of training pixels. When K=1, this decision rule corresponds to the method of minimum distance.

7.3.4 Maximum likelihood method for digital image classification

The maximum likelihood decision rule assigns each pixel of characteristic vector X to the information class C for which the components are more capable of producing the characteristic vector X. The maximum likelihood method for image classification uses the statistics calculated on each class, including the Mean μ_c and the covariance matrix V_c for each class C. This decision rule assigns the vector X to the class C if and only if the following equations are satisfied.

$$p_c(X) = \max_j \left\{ p_j(X) \right\} \quad \Rightarrow \quad X \in w_c \qquad \text{(Eq. 6.9)}$$

$$p_c(X) = \frac{1}{(2\pi)^{n/2} \left| V_c \right|^{1/2}} \exp\left\{ -\frac{1}{2} \left(X - \mu_c \right)^T V_c^{-1} \left(X - \mu_c \right) \right\} \qquad \text{(Eq. 10)}$$

Where $\left| V_c \right|$ represents the Determinant of the covariance matrix V_c.

To assign a characteristic vector X of an unknown pixel to an information class, the classifier calculates the value of $p_c(X)$ for each class. The pixel belongs to the class that maximises this value. The following figure presents an example of a multispectral image classification using the maximum likelihood method.

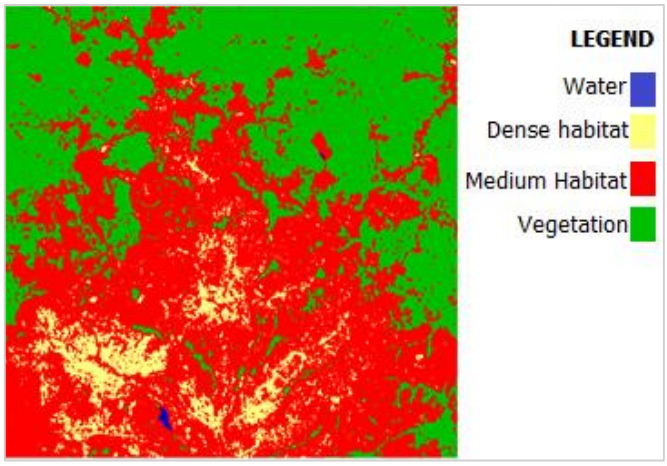


Figure 7.2. Example of Yaoundé town SPOT image classification using the maximum likelihood method.

7.4 Non-supervised classification algorithms

In contrary to supervised classification, the non-supervised classification requires only a minimal quantity of initial inputs. Numerical operations are done to determine natural groupings of spectral properties of pixels in the space of multispectral characteristics.

The classifier calculates the segments' statistics (Mean and covariance matrix) to use for the classification. Once data are classified, the analyst tries to assign these natural or spectral classes to information classes. This task is not ease at all because some segments doesn't have any signification on the field, for example a mixture of different information classes. It is therefore absolutely necessary to better understand the spectral characteristics on the field, to be able to consider certain segments as representing information classes. Various methods of image segmentation have been developed for

multiple applications starting from Pattern Recognition to remote sensing. Segmentation algorithms used for non-supervised classification of remote sensing images don't generally have the same efficiency. This is why various approaches were progressively developed.

7.4.1 Histogram modes method for image classification

A mono-dimensional histogram of an image is a graphical representation of greyscales' frequencies. It is a two-dimensional graph with grayscales on abscises and frequencies on ordinates. A histogram **mode** is a local maximum, while a histogram **valley** is a local minimum of the histogram. A histogram mode indicates a grouping of pixels and helps to detect spectral classes. The greyscale corresponding to a maximum in the histogram is a centre of the spectral class. Following is an algorithm of image classification based on a mono-dimensional histogram.

Algorithm 7.1 Image_Classification_By_Histogram

Input: IMG: unclassified digital image of size NC columns and NL lines;

Output: IMA: classified digital image with the same size as IMG;

Local variable:

K,i,j,n=0: **integer;**

H: Array[0,..,GS_{Max}] of integers;// GS_{Max} is the maximum greyscale in the image;

C: Array[0,..,N] of bytes; // Array of centres of spectral classes;

BEGIN

Determine the maximum greyscale GS_{Max} and the number N of classes of the image IMG;

FOR i varying from 0 to GSMax **DO**

H[i]←0; *//Initialisation of the Histogram;*

FOR j varying from 0 to NL **DO**

FOR i varying from 0 to NC **DO**

H(IMG[j,i])← IMG[j,i]+1; *//Construction of the histogram;*

FOR i varying from 1 to GSMax-1 **DO**

IF(H[i-1]<H[i]>H[i+1]) **THEN**

C[n]←I;

n←n+1;

END_IF

FOR j varying from 0 to NL **DO**

```
    FOR i varying from 0 to NC DO
     BEGIN
        x← IMG[j,i];
        a←C[0];
        FOR k varying from 1 to N DO
         IF (C[k]-x)<(a-x) THEN
           a←C[k];
        END_FOR
        IMA[j,i]←a;
     END_FOR
END
```

The following figure illustrates the algorithm. The original image is acquire on the Cameroonian Atlantic coast.

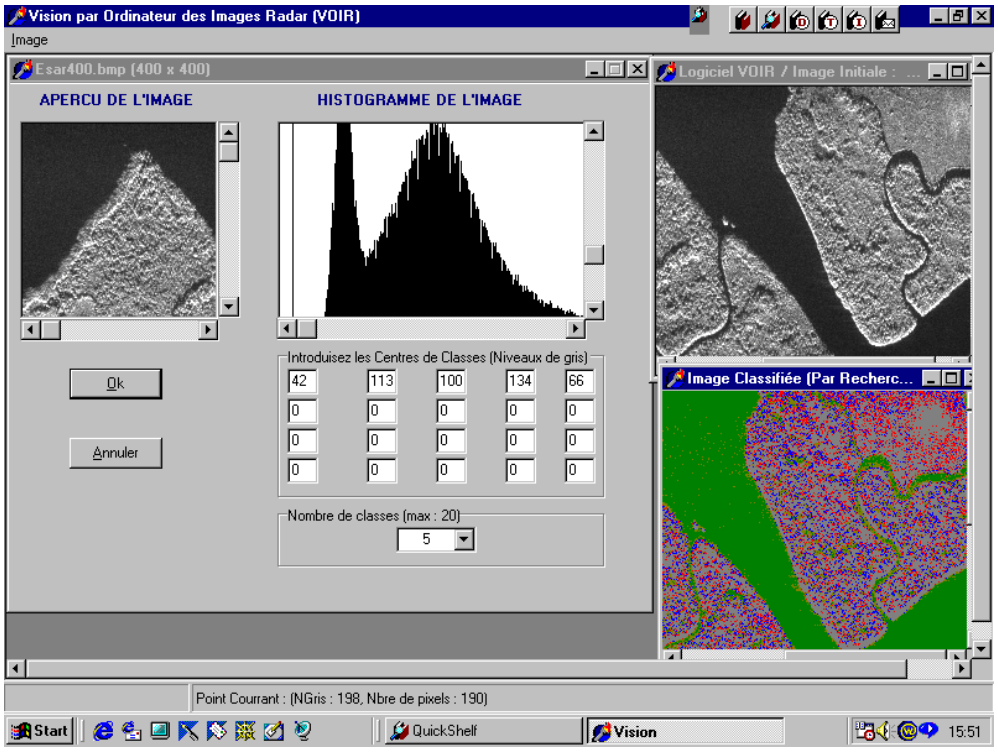


Figure 7.3 Classification of an ESAR image using the histogram mode method.

A multi-band classification can also be done by the means of a bi-dimensional histogram. Such histogram counts the number of occurrences of each pair of greyscales of equivalent pixels in the two bands. Once the bi-dimensional histogram is calculated, the classification algorithm is similar to the previous one.

7.4.2 ISODATA method for non-supervised image classification

The ISODATA (Iterative Self-Organizing Data Analysis Technique) method represents a set of heuristic procedures incorporated in an interactive classification algorithm. This method is auto-organiser because it requires less input from the analyst. A sophisticated ISODATA algorithm requires normally the following criteria.

1) C_{max} : the maximum number of segments to be identified by the algorithm;
2) T: The maximum percentage of pixels for which the class doesn't absolutely change between two successive iterations. The ISODATA algorithm stops when this number is reached.
3) M: The maximum number of time that the ISODATA algorithm classifies pixels and recalculates the average vectors of segments. The ISODATA algorithm stops when this number is reached.
4) The minimum number of members (%) in a segment. If a segment contains less members than this minimum, it is deleted and its members are assigned to another segment.
5) The minimum standard Deviation. When the Standard Deviation of a segment is greater than this minimum and the number of members in the segment twice greater than the specified minimum in a class, the segment is split into two segments.
6) The minimum distance between the segments. Segments with pondered distance less than this value are merged.

The following figure presents an example of a multispectral SPOT image classification using the ISODATA method. The image is acquired on Yaoundé town.

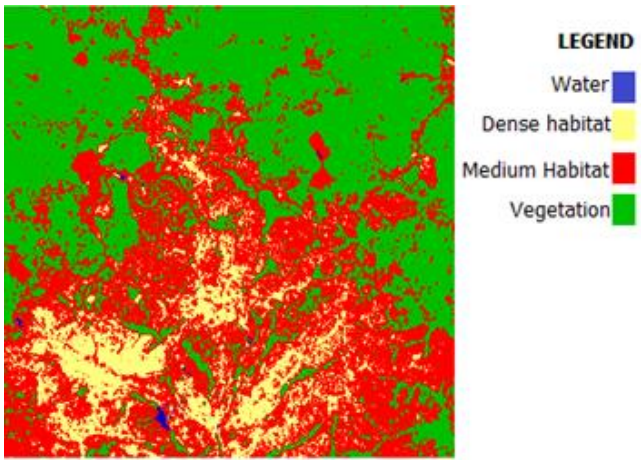


Figure 7.4. Example of Yaoundé town SPOT image classification using the ISODATA method.

7.5 Neuronal networks method for image classification

Neuronal networks are applied in various domains. In image classification, this method is relatively fast, although it requires most of the time a long time for training. Following are some reasons that motivate the use of such a method in remote sensing data processing.

1) It is more accurate than other methods;
2) It has the capacity of processing great data sets faster statistical methods for example;
3) It enables the possibility of integrating a priori knowledge and real physical constraints in the analysis process.
4) It enables the incorporation of different types of data (including data from different sensors) in the analysis process, facilitating the synergy study.

Among neuronal networks methods, the commonly used for remote sensing image classification is the *perceptron multilayers*, trained with the *retro-propagation* algorithm.

7.6 Determination of classification accuracy

The calculation of the classification accuracy consists in comparing the classified image with collected field data. This comparison is usually based on a so-called confusion matrix that indicates the agreements and disagreements between data sets. Measures such as Kappa and well classified percentage derived from the confusion matrix help to express the classification accuracy.

7.6.1 Training and verification data

The classification accuracy is essential for supervised image classification in remote sensing. Typically, the classification accuracy is estimated comparing, for each verification data collected from the field, the information class automatically assigned by the classifier and the effective class on the field. Quantitative indices expressing the accuracy can be calculated from the confusion matrix. The expression of the classification accuracy is tributary to the type of verification data. For a good classification accuracy expression, verification data as training data must be representative of training classes. Consequently, verification data must be collected in verification sites different from training sites from which training data were collected, but belonging to the same information class. Furthermore, data samples must be large to enable a rigorous evaluation of the

classification accuracy. It is also good to collect the verification data at the same moment that training data are collected, this to guarantee that the information class is exactly the same for both training and verification. If possible, these data must be collected within the same period of time as the image acquisition date, because the natural environment changes with time.

7.6.2 Size of samples

The effective number of verification pixels for the estimation of the classification accuracy is difficult to determine. Some analysts use equations based on the proportion of well-classified training data and the admitted classification error. For example, the size N of data used to estimate the accuracy of an image classification can be determine from the following equation:

$$N = \frac{Z^2 p(100 - p)}{E^2} \qquad \text{(Eq. 11)}$$

Where Z is the normal distance apart, with the confidence threshold of 95%. p is the awaited accuracy percentage and E is the admitted error for the estimation of p. The choice of E is normally arbitrary, but most of the time, the value E=4% is adopted.

7.6.3 Sampling methods

Most of robust measures of statistic evaluation of error suppose that reference data are randomly collected. The simple sampling produces suitable estimations of statistical parameters as the sample's size is big. However, random-sampling can under-sample small classes as the sample's size s significantly big. More and more, analysts prefer random sampling by stratum in which a minimum number of samples are selected in each stratum (category of land cover). The best equilibrium between the statistical validity and the practical application consists in combining the random-sampling with the sampling by stratum. Such a system can use a random sampling to collect the validation data at the beginning of the project and use the stratum sampling at the end of the classification process.

7.6.4 Global precision and KAPPA coefficient

The global precision is obtained by splitting the total number of well-classified pixels (sum of the diagonal elements of the confusion matrix) by the total number of verification

pixels. The precision of a particular category is obtained by dividing the total number of well-classified pixels in the category by the total number of pixels of the category. This statistic indicates on the one hand the probability for a pixel of the class to be well classified and the probability of error on the other hand. It is called producer precision because the producer of the classifier is only interested in the success with which a given region in the image can be classified. The commission error is the ratio between the total number of well-classified pixels in the category and the total number of pixels effectively classified in the category. This measure called user precision or reliability is the probability for a pixel classified in a given class represents effectively the assigned class on the field.

KAPA analysis is a discrete and multivariable technique, used to evaluate the classification evaluation. It produces a KHAT (an estimation of KAPPA) which is a measure of agreement or precision. The statistical KHAT parameter is defined by the following equation.

$$\hat{K} = \frac{N\sum_{i=1}^{r} x_{ii} - \sum_{i=1}^{r}\left(x_{i+} \times x_{+i}\right)}{N^2 - \sum_{i=1}^{r}\left(x_{i+} \times x_{+i}\right)} \qquad \text{(Eq. 7.22)}$$

Where r is the number of lines of the confusion matrix. X_{ii} is the number of observations at the point of coordinates (i,i). x_{i+} and x_{+i} are respectively the marginal totals of line i and column I N is the total number of observations.

The global and the KAPA precisions are not usually equal for a given classification. This difference is due to the fact that the global precision uses only diagonal elements of the confusion matrix, ignoring by then omitting and commission errors indicated by the diagonal elements. Nevertheless, KHAT parameter integrates elements out of the diagonal in terms of products of marginal sums of lines and columns. Thus, depending on the number of errors contained in the matrix, it is possible for these two measures to have different values.

CHAPTER 8. REMOTE SENSING IMAGE ANALYSIS

8.1 Introduction

While analysing remote sensing images, photo-interpreters consider the synergy between the context, the contours, the texture and the greyscale variations. But most of classification algorithms are exclusively based on the spectral information (grayscales). This is why it is still a challenge to integrate other characteristics such as texture in classification processes. In this chapter, we develop the notion of texture, fractal analysis, granulometric analysis of an image texture and texture spectrum. We search appropriate combinations of parameters for a given problem in order to optimise the processing time and to reduce the probability of bad classification.

8.2 Texture transformations

8.2.1 Introduction

The texture concept traduces the local homogeneous of an object surface. It is a very useful concept and widely used in various image processing domains. Despite the popularity of this concept, there is no universal definition associated. The texture can be defined as a structure with certain spatial properties, homogeneous and invariant by translation. This definition stipulates that the texture gives the same impression to the observer, independently to the spatial position of the window through which the texture is observed. Obviously, the window's size must be specified. Another definition consisders that the texture concept is linked to three principle concepts:

1) A certain local order that is repeated in a big size region;
2) This order is defined by a structured arrangement of elementary components;
3) These elementary components represent uniform entities, characterised similar dimensions in the hole considered region.

Globally, there are two categories of texture: Structured texture (macro-texture) and random texture (micro-texture). A structured texture is made of a regular repetition of primitives (objects) on a surface. Random structures present a non-organised aspect. From the colour of a pixel, it is not possible to predict the colour of the following colour. In a random texture, in contrary to structured texture, it is not possible to extract a

repeated primitive, but a vector of homogeneous statistic parameters. Since in remote sensing, image textures widely used are random textures, methods based on statistical analysis are probably suitable for image analysis. Following are some famous methods of image analysis based on statistical analysis:

- Random process models,
- Methods for analysis of frequency characteristics,
- Methods for analysis of structural features,
- Methods for analysis of spatial characteristics.

Methods based on random process models consider the spatial dependence of greyscales in an image neighbourhood. The model adequacy and the accuracy of parameters can be verified through a synthesis of analysed texture compared with the original texture.

Methods based on the analysis of spatial characteristics are the commonly used in remote sensing images. These methods can be classified in four groups:

- Generalized histograms of greyscales;
- Measuring the activity of the texture signal;
- Mathematical morphology;
- Fractal analysis.

Generalized histograms measure statistics on greyscales distribution in an image neighbourhood. The size and the orientation of regular structured texture primitives can be obtained through an autocorrelation function or by an order two or three co-occurrence histogram method.

The activity of a texture signal can be estimated by different approaches. Local extrema and contour density can be measured. The curvilinear length of the signal on a given path can also be calculated.

Mathematical morphology has amply demonstrated its effectiveness on organic and metallurgical images. Structuring elements of various sizes and forms help to highlight particular elements on textures. These methods are suitable for structured textures notably.

In the fractal approach, the greyscale amplitude can be assimilated to the altitude of a geometric surface. The fractal dimension of this surface is used to characterise the texture. This method is very effective in the analysis of random texture of type micro-texture or macro-texture, such as Ultrasound lung.

Frequency characteristics measure the spatial frequency components of the texture. Fourier Transform is usually used, based on power spectrum, to calculate these components. Other sophisticated transforms such as the wavelet transform of the Gabor filter have been develop to outperform some limit of Fourier Transform.

8.2.2 Modeling by random process

8.2.2.1. 2-D linear predicting model

This technique supposes that the greyscale of the considered pixel is a linear combination of greyscales of neighbouring pixels. It is now possible to characterise the image texture

$$f = \{IMG(j,i); 0 \le j \le NL-1; 0 \le i \le NC-1\} \quad \text{(Eq. 8.1)}$$

Where NC and NL are respectively the number of columns and lines of the image IMG.

From the previous equation, the image texture can be written in the following form:

$$f(i,j) = \hat{f}(i,j) + \varepsilon(i,j) \quad \text{(Eq. 8.2)}$$

Where $\hat{f}(i,j)$ is the prediction defined as follows:

$$\hat{f}(i,j) = \sum_{(k,l)\in R} \alpha(k,l) \cdot f(i-k, j-l) \quad \text{(Eq. 8.3)}$$

Where $\alpha(k,l)$ is the prediction coefficient; R is the prediction domain and $\varepsilon(i,j)$ is the error between the prediction and the observation.

8.2.2.2. Markov model

This technique supposes that the greyscale of the considered pixel depends on a reduced neighbourhood of the pixel. The Markov process is totally defined by:

- An initial state
- A set of probabilities between states (transition matrix of the process)

Each state of the process corresponds to a particular distribution of greyscales in the neighbourhood. Markov random fields can be used for texture analysis and synthesis. It can also be used for textured images segmentation. Texture analysis with Markov model consists in estimating the parameters of the model which characterise the texture.

It is demonstrated that texture modelling parameters are spatial frequencies. These parameters are generally estimated in four principal directions: 0°, 45°, 90° and 135°. A Markov field is a dependence model that exists between image greyscales. The principal property of Markov field stipulates that the value of greyscale doesn't depend on

greyscales of all the image, but only depends on greyscales of some of them that belong to the neighbourhood of the considered pixel. It is admitted that a pixel doesn't belong to its neighbourhood. Furthermore, if a pixel P1 belongs to the neighbourhood of a pixel P2, then P2 belongs to the neighbourhood of P1. The following pixel presents an example of 4-neighbourhood of a pixel (i,j), made of the 4 nearest neighbouring pixels.

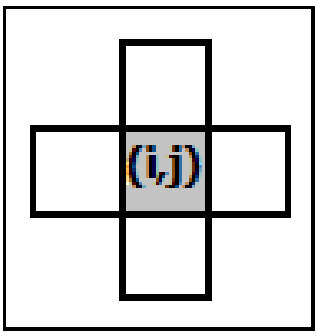


Figure 8.2 Example of a 4-neighbourhood of a pixel (i,j).

Since the value of a pixel depends only on the nearest neighbouring pixels, it is useful to study this dependence in order to understand how the parameters of the Markov model behave with respect to the relationship that exists between neighbouring pixels. For this purpose, we introduce the notion of Clique. A clique is a set of one, two, three or more pixels associated to a previously defined 4-neighbourhood, 8-neighbourhood or other neighbourhood, organised such that each pixel belongs to the neighbourhood of others.

The following figure presents an example of a 4-neighbourhood with the associated cliques. One can distinguish one clique made of one pixel (singleton) and 4 cliques made of two pixels (doubleton).

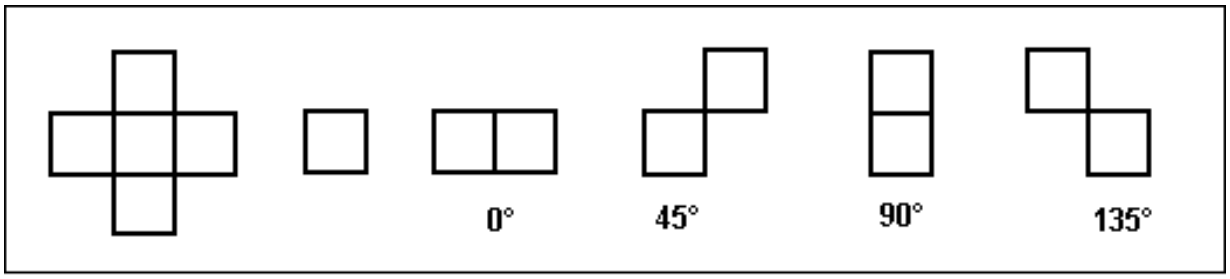


Figure 8.3 A 4-neighbourhood and associated cliques.

To each clique is associated a function called « potential function » which is a linear function of the parameter corresponding to the clique. The value of the potential function depends on the relationship between the greyscales of the pixels in the clique. Estimated values of the clique hep to determine the spatial frequencies of the greyscales in the given direction. In most models, spatial frequencies increase as values of parameters decrease. Markov models are widely used in noisy or textured images segmentation.

8.2.3. Methods based on spatial characteristics

8.2.3.1. Generalised histograms of greyscales

The textural information associated to a pixel is contained in the spatial relations between the pixel and its neighbours. This information can be statistically characterised by an N-dimensional histogram in which each entry $p\{f(i_1), f(i_2), \ldots\ldots, f(i_n)\}$ represents an estimate of the probability of $\{f(i_1), f(i_2), \ldots\ldots, f(i_n)\}$; N being the number of neighbours involved and $f(i_k)$ the greyscale of the pixel i_k. From these multi-dimensional histograms, various statistics of different orders can be calculated to characterise the image texture.

8.2.3.2. First order histogram-based method

First order histogram is widely known under the name "Image histogram". It describes the distribution of greyscales in the image. Each entry i of the histogram defines the frequency p(i) of the greyscale i in the image. Such a histogram is not interested in the spatial relationship that may exists between a pixel and its neighbour, but only on the value of the pixel's greyscale. A first order histogram can be used for a segmentation by thresholding on greyscales in case the mage contains uniform zones. Various statistics of first order such as the Variance or the Mean can be calculated from a first order histogram of greyscales. Following are some commonly used statistics.

- ***Mean of greyscales (μ)***: $W_1 = \sum_{i=0}^{GS\,max} i \times P(i)$ (Eq. 8.4)

The Mean represents the positioning of the histogram on the scale of greyscales.

- ***Entropy:*** $W_2 = -\sum_{i=0}^{GS\,max} P(i) \times \ln[P(i)]$ (Eq. 8.5)

The entropy parameter measures the degree of disorder in a texture.

- **Energy:** $W_3 = \sum_{i=0}^{GS\,max} [P(i)]^2$ (Eq. 8.6)

The Energy parameter increases with the homogeneity of the texture.

- **Variance of greyscales (σ^2):** $W_4 = \sum_{i=0}^{GS\,max} (i-\mu)^2 P(i)$ (Eq. 8.7)

The variance is a measure of the variability of greyscales around the Mean.

- **Symmetry coefficient:** $W_5 = \frac{1}{\sigma^3}\left[\sum_{i=0}^{GS\,max} (i-\mu)^3 P(i)\right]$ (Eq. 8.8)

The symmetry coefficient measures the symmetry of the distribution of greyscales around the Mean.

- **Flatten coefficient:** $$W_6 = \frac{1}{\sigma^4}\left[\sum_{i=0}^{GS\,max}(i-\mu)^4 P(i)\right] \quad \text{(Eq. 8.9)}$$

The Flatten coefficient gives an indication on the aspect (flatten or not) of the histogram.

- **Variation coefficient:** $$W_7 = \frac{\sigma}{\mu}\times 100\% \quad \text{(Eq. 8.10)}$$

The variation coefficient measures the local heterogeneity of the image.

8.2.3.3. Second order histogram (co-occurrence matrix) method

Let $c = (\Delta x, \Delta y)$ be a vector in the image plan, where Δx and Δy are integers representing the displacement respectively on x and y-coordinates. From a point (x_1,y_1), with a displacement $(\Delta x, \Delta y)$, the ending point is $(x_2,y_2)=(x_1+\Delta x, y_1+\Delta y)$. This displacement can be defined as a couple (d, θ), where d is the inter-pixel distance and θ is the inter-pixel orientation. Commonly, the value of θ is choosen between 0°, 45°, 90° and 135°. This displacement defines the coordinates of the first pixel i and the following pixel j involved in the expression of a parameter. A second order textural parameter is a function of the second order histogram called co-occurrence matrix. The co-occurrence matrix is a bi-dimensional usually calculated on mage windows extracted from the image. An entry (i, j) of the co-occurrence matrix expresses the number of times that a pixel with greyscale i is followed by a pixel with greyscale j, considering the displacement $(\Delta x, \Delta y)$ in the image window. This number, divided by the total number of couples in the image window with respect to the displacement, gives the probability of occurrence the couple of greyscales (i, j) with respect to the considered displacement. If the maximum greyscale n the image is GS_Max, then the co-occurrence matrix should be a bi-dimensional array of size GS_Max+1. Let consider for example the following image window of size 5x5 and let calculate the related co-occurrence matrix.

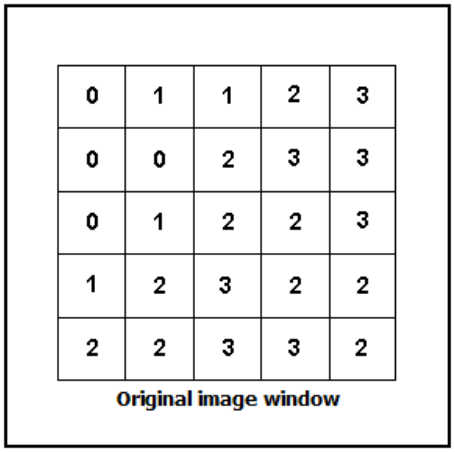


Figure 8.4. image window for which the co-occurrence matrix is calculated.

Since the maximum greyscale in the image window is 3, the co-occurrence matrix should be of size 4x4. Following are the co-occurrence matrices for various displacements.

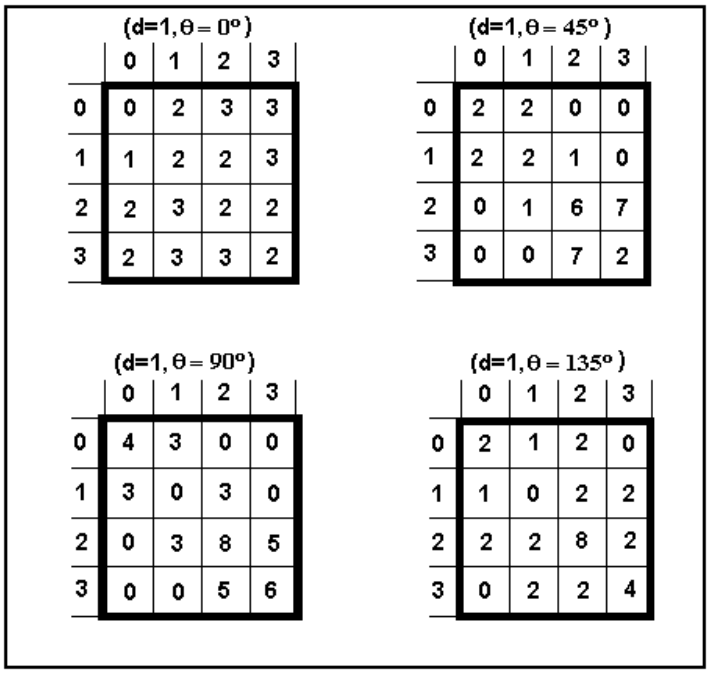


Figure 8.5 Co-occurrence Matrices calculated from the image window of figure 8.4.

Following are some textural parameters defined to characterise textural information from the co-occurrence matrices.

- ***Second Angular Moment (Energy):*** $$SAM = \sum_{i=0}^{GS_{max}} \sum_{j=0}^{GS_{max}} [p(i,j)]^2 \qquad \text{(Eq. 8.11)}$$

The Second Angular Moment (SAM) measures the homogeneity of the image. Its value is weak when the values of p(i,j) are nearer. In contrary, if some values of p(i,j) are big while others are small then its value should be high. This situation happens when the values of the co-occurrence matrix are concentrated around the diagonal.

❖ **Contrast (Inertia):** $$CST = \sum_{i=0}^{GS_{max}} \sum_{j=0}^{GS_{max}} (i-j)^2 p(i,j) \quad \text{(Eq. 8.12)}$$

The contrast is a measure of the local variation of image greyscales. In contrary to the Energy parameter, the value of the contrast is great when the values of the co-occurrence matrix are concentrated out of the diagonal. So, the contrast increases (resp. decreases) as the second angular moment decreases (resp. increases).

❖ **Correlation:** $$COR = \sum_{i=0}^{GS_{max}} \sum_{j=0}^{GS_{max}} \frac{(i-\mu_x)(j-\mu_y)}{\sigma_x \sigma_y} p(i,j) \quad \text{(Eq. 8.13)}$$

Where μ_x, μ_y, σ_x, and σ_y are respectively the Mean and Standard Deviation with respect to the x and y-coordinates.

Marginal distributions with respect to x and y-coordinates, associated to the co-occurrence matrix, are defined as follows:

$$\begin{cases} p_x(i) = \sum_{j=0}^{GS_{max}} p(i,j) \\ p_y(j) = \sum_{i=0}^{GS\,max} p(i,j) \end{cases} \quad \text{(Eq. 8.14)}$$

This parameter is the correlation coefficient of pixels of the image window with a linear model, the regression line being the diagonal of the co-occurrence matrix. The value of this parameter is high if and only if the values of the co-occurrence matrix are uniformly distributed.

❖ **Sum of variance squares:** $$VAR = \sum_{i=0}^{GS_{max}} \sum_{j=0}^{GS_{max}} (i-\mu_x)^2 p(i,j) \quad \text{(Eq. 8.15)}$$

❖ **Inverse Differential Moment:** $$IDM = \sum_{i=0}^{GS_{max}} \sum_{j=0}^{GS_{max}} \frac{1}{1+(i-j)^2} p(i,j) \quad \text{(Eq. 8.16)}$$

❖ **Sum of Averages:** $$SAVG = \sum_{i=2}^{2GS_{max}} i \times p_{x+y}(i) \quad \text{(Eq. 8.17)}$$

❖ **Sum of Variances:** $$SVAR = \sum_{i=2}^{2GS_{max}} (i-ENT)^2 p_{x+y}(i) \quad \text{(Eq. 8.18)}$$

❖ **Entropy:** $$ENT = -\sum_{i=0}^{GS_{max}} \sum_{j=0}^{GS_{max}} p(i,j) \log(p(i,j)) \quad \text{(Eq. 8.19)}$$

❖ **Sum of Entropies:** $$SENT = -\sum_{i=2}^{2GS_{max}} p_{x+y}(i) \log\left(p_{x+y}(i)\right) \quad \text{(Eq. 8.20)}$$

The entropy measures the complexity of the image. Its value is high if and only if the values of the co-occurrence matrix are almost equal. Otherwise, it has a low value.

- **Variance of Difference:** $VDIF = \text{variance} \quad of \quad p_{x-y}$ (Eq. 8.21)
- **Entropy of Diference:** $EDIF = -\sum_{i=2}^{2GS_{\max}} p_{x-y}(i) \times \log\left(p_{x-y}(i)\right)$ (Eq. 8.22)
- **Measure of correlation information:**
  - $MCI_1 = \dfrac{HXY - HXY1}{\max\{HX, HY\}}$ (Eq. 8.23)
  - $MCI_2 = \left[1 - \exp\left(-2\left(HXY2 - HXY\right)\right)\right]^{1/2}$ (Eq. 8.24)

Where:

HX and HY are entropies of P_x and P_y respectively.

$$HXY = -\sum_{i=0}^{GS_{\max}} \sum_{j=0}^{GS_{\max}} p(i,j)\log\left(p(i,j)\right), \quad \text{(Eq. 8.25)}$$

$$HXY1 = -\sum_{i=0}^{GS_{\max}} \sum_{j=0}^{GS_{\max}} p(i,j) \times \log\left(p_x(i)p_y(j)\right), \quad \text{(Eq. 8.26)}$$

$$HXY2 = -\sum_{i=0}^{GS_{\max}} \sum_{j=0}^{GS_{\max}} p_x(i)p_y(j) \times \log\left(p_x(i)p_y(j)\right). \quad \text{(Eq. 8.27)}$$

Maximal coefficient of correlation:

MaxCOR=[Second greatest value of Q]$^{1/2}$ (Eq. 8.28)

Where:

$$Q(i,j) = \sum_k \frac{p(i,k)p(j,k)}{p_x(i)p_y(k)}, \; k = 0,1,2,3,\ldots\ldots,GS_{\max} \quad \text{(Eq. 8.29)}$$

$$p_{x+y}(k) = \sum_{i=0}^{GS_{\max}} \sum_{j=0}^{GS_{\max}} \{p(i,j); |i+j| = k\}, \; k = 2,3,\ldots\ldots,2GS_{\max} \quad \text{(Eq. 8.30)}$$

$$p_{x-y}(k) = \sum_{i=0}^{GS_{\max}} \sum_{j=0}^{GS_{\max}} \{p(i,j); |i-j| = k\}, \; k = 0,1,2,3,\ldots\ldots,GS_{\max} \quad \text{(Eq. 8.31)}$$

- **Diagonal Moment:** $DM = \sum_{i=0}^{GS_{\max}} \sum_{j=0}^{GS_{\max}} \left(\frac{1}{2}|i-j|p(i,j)\right)^{1/2}$ (Eq. 8.32)
- **Mean (Average):** $AVG = \sum_{i=0}^{GS_{\max}} \sum_{j=0}^{GS_{\max}} i \times p(i,j)$ (Eq. 8.33)
- **Cluster Shade:** $CS = \sum_{i=0}^{GS_{\max}} \sum_{j=0}^{GS_{\max}} (i+j-2AVG)^3 p(i,j)$ (Eq. 8.34)

- **Cluster Proeminence:** $CP = \sum_{i=0}^{GS_{max}} \sum_{j=0}^{GS_{max}} (i + j - 2AVG)^4 \, p(i,j)$ (Eq. 8.35)
- **Inverse Difference:** $INVD = \sum_{i=0}^{GS_{max}} \sum_{j=0}^{GS_{max}} \frac{p(i,j)}{1+|i-j|}$ (Eq. 8.36)
- **Dissimilarity (Absolute Value):** $DISS = \sum_{i=0}^{GS_{max}} \sum_{j=0}^{GS_{max}} |i-j| p(i,j)$ (Eq. 8.37)
- **Covariance:** $COV = \sum_{i=0}^{GS_{max}} \sum_{j=0}^{GS_{max}} (i-\mu_x)(j-\mu_y) p(i,j)$ (Eq. 8.38)
- **Maximal Probability:** $PMax = \max_{0 \le i,j \le GS_{max}} \{p(i,j)\}$ (Eq. 8.39)
- **Third Angular Moment:** $TAM = \sum_{i=0}^{GS_{max}} \sum_{j=0}^{GS_{max}} [p(i,j)]^3$ (Eq. 8.40)

Where GS_{max} is the maximum greyscale in the image and *p* the probability of occurrence of each couple of greyscales with respect to the adopted displacement. This probability is defined by: $p(i,j) = \frac{1}{N} CM(i,j)$, where N is the total number of possible couples, with respect to the displacement, in the image. Let define the values of N with respect to the principal inter-pixel orientations and a generic inter-pixel distance d.

	Θ=0°	Θ=45°	Θ=90°	Θ=135°
N(d, Θ)	$(W_x-d)*W_y$	$(W_x-d)*(W_y-d)$	$W_x*(W_y-d)$	$(W_x-d)*(W_y-d)$

Most of these parameters has similar properties with the first ones. But some of them give further information. This is why it is necessary to select suitable parameters before starting the process of the image. Obviously, no parameter is strictly better than another one. But for a given application and a given image, a parameter can be more adapted than another one.

8.2.4. Algorithm for image texture calculation

An image texture is the result of an application of a textural parameter by the mean of image convolution. For this purpose, the displacement (inter-pixel distance d and orientation Θ must be defined), so as the size of the image window to use. Following is the related algorithm.

Algorithm 8.1. Image texture calculation

Input: IMG: Original image of NC columns and NL lines;

W_x, W_y: Integers representing the number of columns and lines respectively;

d,Θ: displacement (inter-pixel distance and orientation);

Para(W,d,Θ): Expression of the texture parameter for an image window W;

Border filling method (zero, symmetry or circular symmetry method)

Output: IMA: Image texture;

Local Variables: i, j: Integers;

GS_{max}: integer representing the maximum greyscale in the image;

P: Co-occurrence matrix=Array[0..GS_{max};0..GS_{max}] of integers;

BEGIN

FOR j varying from 0 to NL, **DO**

FOR i varying from 0 to NC, **DO**

BEGIN

Extract an image window W centred on (j,i), using the adopted border filling method for border points;

Calculate the co-occurrence matrix P for the image window W, with respect to the inter-pixel distance d and orientation Θ;

Calculate the texture parameter Param(W,d,Θ) for the image window W;

Assign Param(W,d,Θ) to IMA(j,i)

END;

Normalise IMA;

END

8.2.5 Example of image texture calculation

Let consider the following experimental image IMG with the displacement (d,Θ)=(1,45°), and image window of size 3x3 and the Zero method for border filling.

IMG=

0	1	2	4	3
4	0	0	2	3
4	4	2	0	1
4	3	2	1	2
4	2	4	4	4

Figure 8.6. An experimental Image

Let first calculate the texture parameter for the central point.

8.2.5.1: Image window extraction

Following is the image window extraction:

$$W = \begin{pmatrix} 0 & 0 & 2 \\ 4 & 2 & 0 \\ 3 & 2 & 1 \end{pmatrix}$$

Figure 8.7. Image window centered on the pixel (2,2) of the experimental image.

8.2.5.2: Calculation of the co-occurrence matrix

The co-occurrence matrix is the following:

$$CM = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The total number of possible couples that match with the displacement (d,θ)=(1,45°) is $N=(W_x-1)*(W_y-1)=4$.

8.2.5.3: Calculation of the texture parameter for extracted image window

The expression of the Dissimilarity parameter is the following:

$$\text{Diss} = \sum_{i=0}^{4}\sum_{j=0}^{4}|i-j|P_{ij} \qquad = \frac{1}{2N}\sum_{i=0}^{4}\sum_{j=0}^{4}|i-j|CM_{ij} \qquad = 7/4$$

8.2.5.4: Calculation of the image texture

For each pixel (i,j) of the experimental image, we extract an image window W of size 3x3 centered on (i,j). We calculate the co-occurrence matrix CM and we deduce the value of the texture parameter. The following result is then obtained.

$$1/4\begin{pmatrix} 3 & 6 & 9 & 11 & 11 \\ 11 & 13 & 10 & 8 & 8 \\ 12 & 9 & 7 & 5 & 6 \\ 9 & 2 & 6 & 7 & 8 \\ 11 & 7 & 11 & 13 & 10 \end{pmatrix}$$

8.2.5.5. Normalisation of the image texture

The maximum greyscale in the original image is 4 and the maximum greyscale in the texture image is 13/4. Each term of the texture image should be multiplied by the ratio 16/13 and we obtain the following result:

$$IMA = {}^{16}\!/_{13}\begin{pmatrix} 3 & 6 & 9 & 11 & 11 \\ 11 & 13 & 10 & 8 & 8 \\ 12 & 9 & 7 & 5 & 6 \\ 9 & 2 & 6 & 7 & 8 \\ 11 & 7 & 11 & 13 & 10 \end{pmatrix}$$

8.2.6 Third order statistical parameters

The third order statistical parameter is based on the third order histogram called third order co-occurrence matrix. A third order co-occurrence matrix is a tree-dimensional array for which each entry P(i,j,k) is defined with respect to two displacements. So $p(i,j,k) = p(i,j,k;d_1,\theta_1,d_2,\theta_2)$ meaning that the pixel of greyscale j is separated from the pixel of greyscale i by an inter-pixel distance d_1 in the orientation θ_1 and the pixel of greyscale k is separated from the pixel of greyscale j by an inter-pixel d_2 in the orientation θ_2. This entry expresses the number of occurrences of such a triplet. Based on this co-occurrence matrix, various parameters can be defined.

- **Inverse Diference:**

$$INVD = \sum_{i=0}^{GS\max}\sum_{j=0}^{GS\max}\sum_{k=0}^{GS\max}\frac{p(i,j,k)}{1+|i-j|+|i-k|+|j-k|} \qquad \text{(Eq. 8.41)}$$

- **Dissimilarity or Absolute Value:**

$$DISS = \sum_{i=0}^{GS\max}\sum_{j=0}^{GS\max}\sum_{k=0}^{GS\max}\left[|i-j|+|i-k|+|j-k|\right]\times p(i,j,k) \qquad \text{(Eq. 8.42)}$$

- **Entropy**

$$ENT = \sum_{i=0}^{GS\max}\sum_{j=0}^{GS\max}\sum_{k=0}^{GS\max}p(i,j,k)\times Log\left[p(i,j,k)\right] \qquad \text{(Eq. 8.43)}$$

- **Contrast**

$$CST = \sum_{i=0}^{GS\max}\sum_{j=0}^{GS\max}\sum_{k=0}^{GS\max}\left[(i-j)^2+(i-k)^2+(j-k)^2\right]\times p(i,j,k) \qquad \text{(Eq. 8.44)}$$

- **Second Angular Moment**

$$SAM = \sum_{i=0}^{GS\max}\sum_{j=0}^{GS\max}\sum_{k=0}^{GS\max}\left[p(i,j,k)\right]^2 \qquad \text{(Eq. 8.45)}$$

❖ **Inverse Differential Moment**

$$IDM = \sum_{i=0}^{GS\max}\sum_{j=0}^{GS\max}\sum_{k=0}^{GS\max}\frac{p(i,j,k)}{1+(i-j)^2+(i-k)^2+(j-k)^2} \quad \text{(Eq. 8.46)}$$

❖ **Correlation**

$$COR = \sum_{i=0}^{GS\max}\sum_{j=0}^{GS\max}\sum_{k=0}^{GS\max}\frac{(i-\mu_i)(j-\mu_j)(k-\mu_k)}{\sigma^2}\times p(i,j,k) \quad \text{(Eq. 8.47)}$$

❖ **Covariance**

$$COV = \sum_{i=0}^{GS\max}\sum_{j=0}^{GS\max}\sum_{k=0}^{GS\max}(i-\mu_i)(j-\mu_j)(k-\mu_k)\times p(i,j,k) \quad \text{(Eq. 8.48)}$$

❖ **Variance**

$$VAR = \sum_{i=0}^{GS\max}\sum_{j=0}^{GS\max}\sum_{k=0}^{GS\max}(i-\mu_i)\times p(i,j,k) \quad \text{(Eq. 8.49)}$$

❖ **Maximal Probability**

$$MaxP = \max_{0\le i,j,k\le GS\max}\{p(i,j,k)\} \quad \text{(Eq. 8.50)}$$

❖ **Third Angular Moment**

$$TAM = \sum_{i=0}^{GS\max}\sum_{j=0}^{GS\max}\sum_{k=0}^{GS\max}[p(i,j,k)]^3 \quad \text{(Eq. 8.51)}$$

❖ **Importance of small numbers**

$$ISN = \sum_{i=0}^{GS\max}\sum_{j=0}^{GS\max}\sum_{k=0}^{GS\max}\frac{p(i,j,k)}{(i^2+j^2+k^2)} \quad \text{(Eq. 8.52)}$$

❖ **Importance of great numbers**

$$IGN = \sum_{i=0}^{GS\max}\sum_{j=0}^{GS\max}\sum_{k=0}^{GS\max}(i^2+j^2+k^2)\times p(i,j,k) \quad \text{(Eq. 8.53)}$$

❖ **Importance of deepness**

$$ID = \frac{\sum_{i=1}^{GS\max}\sum_{j=1}^{GS\max}\left(\sum_{k=1}^{GS\max}p(i,j,k)\right)^2}{\sum_{i=1}^{GS\max}\sum_{j=1}^{GS\max}\sum_{k=1}^{GS\max}p(i,j,k)} \quad \text{(Eq. 8.54)}$$

The value of σ^2 is given by:

$$\sigma^2 = \sum_{i=0}^{GS\max}\sum_{j=0}^{GS\max}\sum_{k=0}^{GS\max}(i-\mu_i)(j-\mu_j)(k-\mu_k)\times p(i,j,k) \quad \text{(Eq. 8.55)}$$

Researchers don't often use this method in remote sensing because of time and memory space consuming.

8.3 Vegetation indices

Derived from remote sensing data, vegetation indices constitute a basic and precious information for environment management. Accurate and opportune data collection on world crops are constant preoccupation of human being. Collecting such data by in situ techniques is onerous and time consuming when it is not simply impossible.

One approach consists in measuring the vegetation quantity and condition by multispectral data analysis. Many studies in this area, using multispectral data, have been developed, notably: Landsat MSS (**Multispectral Scanner**), Landsat TM (**Thematic Mapper**), SPOT-HRV (High Visible Resolution) or NOAA-AVHRR (**Advanced Very High Resolution Radiometer**). The aim often consists in transforming various bands into a single image in which the value of a pixel predicts and measures environment characteristics, such as the biomass, the productivity (photo mass) the Leaf Area Index (LAI), the quantity of photosynthetic active radio (PAR) consumed or the percentage of vegetation cover.